



BANK OF ENGLAND

THE LONDON SCHOOL  
OF ECONOMICS AND  
POLITICAL SCIENCE ■

# Taking regulation seriously: Fire sales under solvency and liquidity constraints

Jamie Coen,<sup>1,2</sup> Caterina Lepore<sup>2</sup> and Eric Schaanning<sup>3</sup>  
London School of Economics,<sup>1</sup> Bank of England,<sup>2</sup> European Systemic  
Risk Board<sup>3</sup>

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# Disclaimer

The views expressed are those of the authors only and do not necessarily reflect those of the Bank of England or the European Systemic Risk Board.

① Introduction

② Model

③ Data

④ Results

⑤ Conclusions

⑥ Appendix

# Motivation

- Liquidity issues during the crisis
- Multiple regulatory constraints under Basel III
- Macroprudential stress tests

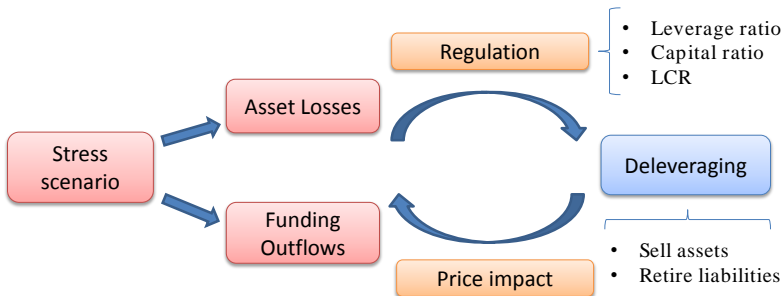
# Objectives

Build a quantitative model of fire sales to assess:

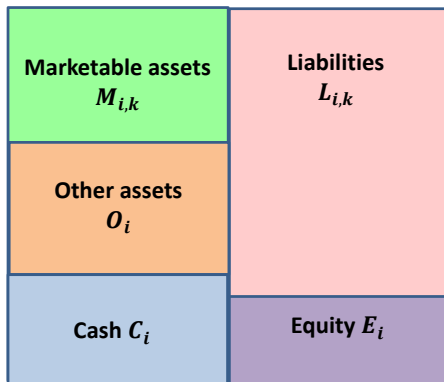
Which types of financial shocks and regulatory requirements combine to produce fire sales?

How do banks optimally liquidate their portfolios when they are forced to do so?

# Model overview



# Bank balance sheets



## Price evolution

$$P_{t+1}^k = P_t^k \left( 1 - \delta_k^{-1} \sum_{i=1}^N S_t^{i,k} \right),$$

where

$$\delta_k = c \frac{ADV_k}{\sigma_k} \sqrt{\tau},$$

- $ADV_k$  is the average trading volume,
- $\sigma_k$  is the daily volatility,
- $\tau$  is the liquidation horizon,
- $c$  is a scaling parameter close to 0.3 ([Obizhaeva, 2012]).

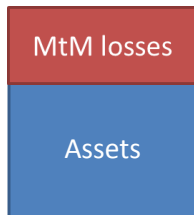


## Fire-sale losses

Two forms of loss:

- Mark-to-market losses

$$\sum_{k=1}^K \underbrace{(M_t^{i,k} - S_t^{i,k})}_{\text{Remaining holdings}} \times \delta_k^{-1} \underbrace{\sum_{i=1}^N S_t^{i,k}}_{\text{Price impact}}$$



## Fire-sale losses

- Implementation shortfall

$$\alpha \sum_{k=1}^K S_t^{i,k} \sum_{j=1}^N \delta_k^{-1} S_t^{j,k}$$



## Fire-sale losses

- Banks only internalise the price impact of their own sales:

$$\frac{S_t^{i,k}}{\delta_k}$$

- and not the effects of sales by other banks:

$$\frac{\sum_{i=1}^N S_t^{i,k}}{\delta_k}.$$

# Banks' deleveraging

- When necessary, banks use the sale proceeds to retire liabilities ( $R^i$ ) in order to either pay out runing depositors or to improve their leverage and/or LCR constraints.
- Banks use the proceeds to retire liabilities in a pecking order of most to least harshly treated by the LCR.

## Bank optimisation: Minimize liquidation losses

$$\min_{\mathbf{S}, \mathbf{R}} (M - \mathbf{S})^\top \delta^{-1} \mathbf{S} + \alpha \mathbf{S}^\top \delta^{-1} \mathbf{S},$$

## Bank optimisation: Minimize liquidation losses

$$\min_{\mathbf{S}, \mathbf{R}} (M - \mathbf{S})^\top \delta^{-1} \mathbf{S} + \alpha \mathbf{S}^\top \delta^{-1} \mathbf{S},$$

subject to the constraints

$$CAP(A, E; \mathbf{S}) \geq REG_{CAP}$$

$$LEV(A, C, E; \mathbf{S}, \mathbf{R}) \geq REG_{LEV}$$

$$LCR(A, C, L; \mathbf{S}, \mathbf{R}) \geq REG_{LCR}$$

$$CASH(A, C; \mathbf{S}, \mathbf{R}) \geq 0.$$

## Bank optimisation: Minimize liquidation losses

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## Optimal liquidation strategy under the leverage ratio

Banks constrained by the leverage ratio prefer to sell first liquid assets of which they do not hold large amounts.

### Proposition

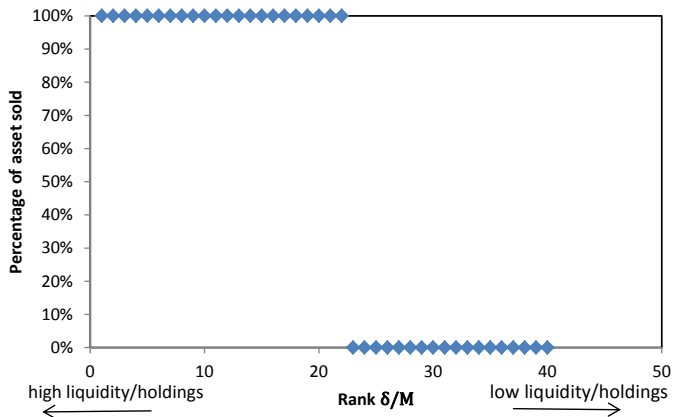
*In the case of full liquidation impact and small liquidation volumes, the optimal strategy is to sell assets sequentially in decreasing order of the ratio:*

$$\frac{\delta_k}{M^k}$$

*until the constraint is satisfied.*



## Asset sales: leverage ratio

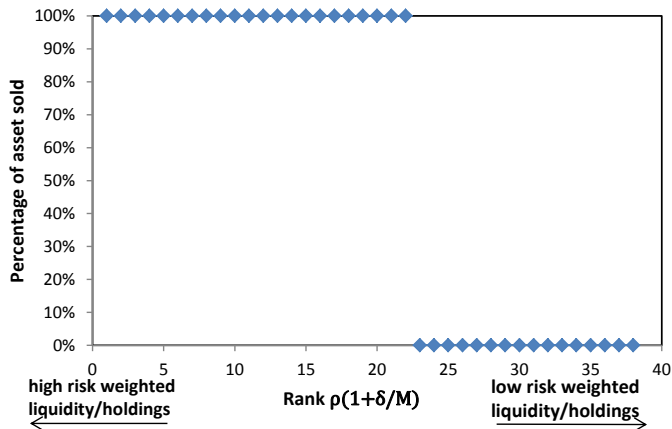


## Optimal liquidation strategy under the capital ratio and liquidity coverage ratio

Banks constrained by the capital ratio and the LCR must balance the liquidity of an asset with its weight in these two regulatory ratios.

- The optimal strategy is to sell assets sequentially in decreasing order of the liquidity to holdings ratio, weighted by the regulatory weights.
- For two assets with the same liquidity to holdings ratio, banks will prefer to sell the one with the higher risk-weight or LCR haircut.

## Asset sales: capital ratio



# Calibration

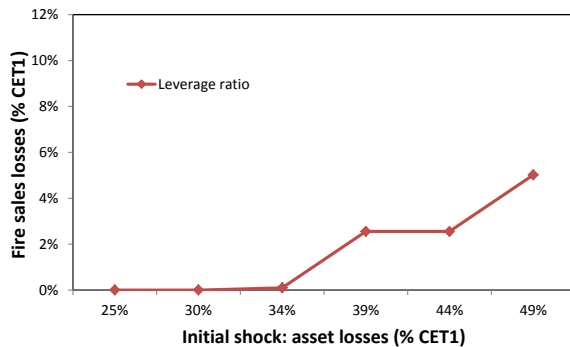
- 7 banks subject to the Bank of England stress test.
- Balance sheet data taken from regulatory returns (COREP and FINREP) and Bank of England stress test data.
- Market depths based on national authorities' published statistics on average trading volumes and outstanding amounts and S&P price indices.

# Stress scenarios

- ① Solvency shock: Bank of England 2017 Stress scenario and variants thereof.
- ② Funding shock: Liquidity Coverage Ratio outflows and variants thereof.
- ③ Combined solvency and funding shocks.

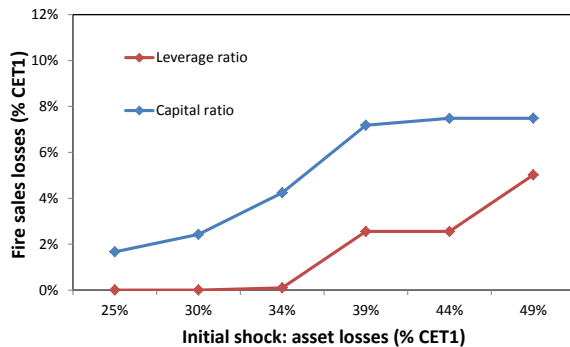
Solvency shocks: variants of 2017 Bank of England stress test

## Fire-sale losses under the leverage ratio



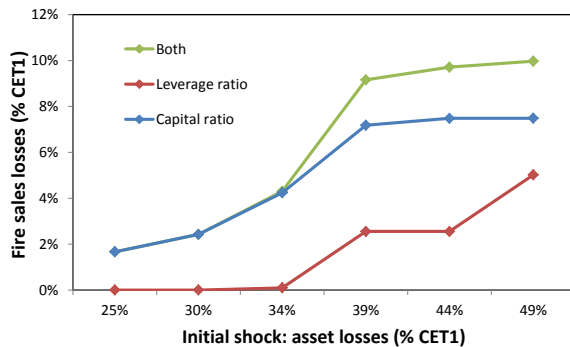
Solvency shocks: variants of 2017 Bank of England stress test

## Fire-sale losses under the capital ratio



Solvency shocks: variants of 2017 Bank of England stress test

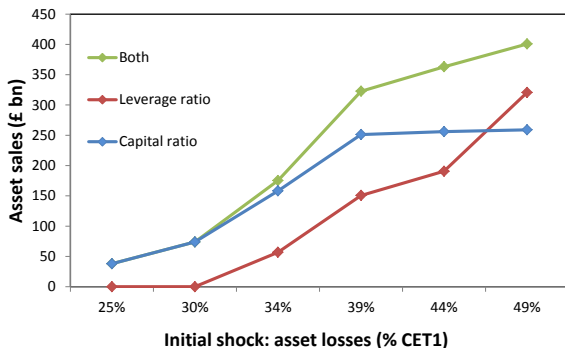
## Fire-sale losses under both solvency constraints





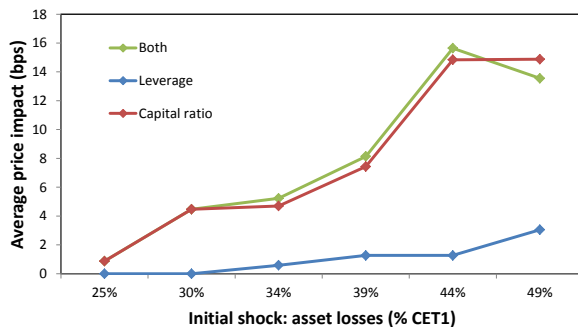
## Solvency shocks: variants of 2017 Bank of England stress test

## Banks sell larger amount of assets under the capital ratio



Solvency shocks: variants of 2017 Bank of England stress test

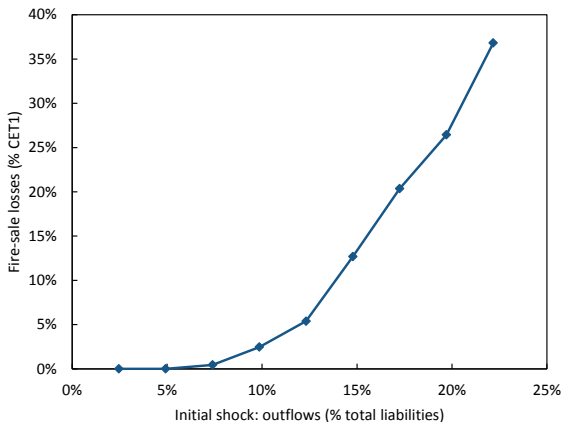
## Banks sell less liquid assets under the capital ratio



# Funding shock

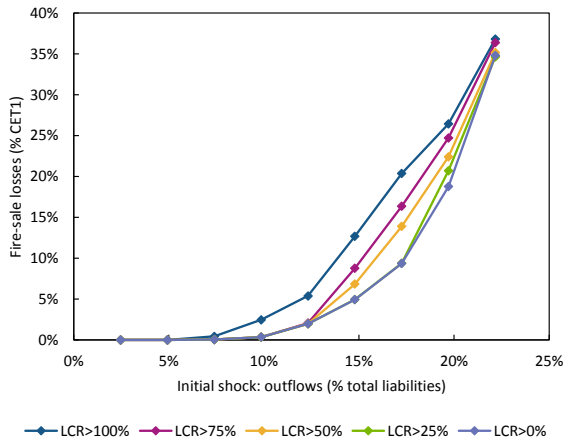
- LCR assigns outflow rates to each category of liability, e.g. 100% for certain types of short-term wholesale funding, 5% for retail deposits.
- We assume creditors run in proportion to these outflow rates.

## Fire-sale losses when banks defend LCR > 100%

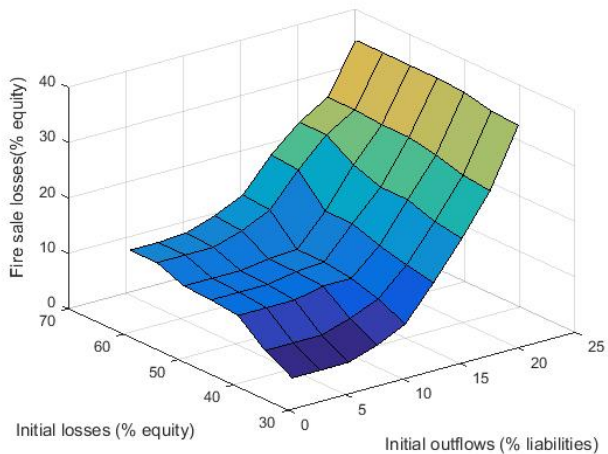


## Funding shocks: variants of Liquidity Coverage Ratio outflows

# Fire-sale losses when banks are willing to let their LCR fall below 100%



# Solvency and funding shocks



# Solvency and funding shocks

Composite funding and solvency shocks result in fire-sale losses that are:

- **Lower** than the sum of the losses following the equivalent funding and solvency shocks in isolation.
- ⇒ Banks' responses to solvency and liquidity shocks are complementary.

## Solvency and funding shocks

Composite funding and solvency shocks result in fire-sale losses that are:

- **Lower** than the sum of the losses following the equivalent funding and solvency shocks in isolation.  
⇒ Banks' responses to solvency and liquidity shocks are complementary.
- **Greater** than either a funding shock or a solvency shock in isolation.  
⇒ Banks' vulnerability and responses to these two type of shocks are heterogeneous.



# Conclusions

- Both risk-weighted capital and LCR constraints incentivise sales of larger amounts of less liquid assets relative to the leverage ratio.
- Fire sales losses due to solvency shocks are relatively small even for extremely large shocks.
- Severe but plausible funding shocks can generate significant fire losses.

# Conclusions

- Stress tests focused exclusively on solvency may underestimate the impact and extent of fire-sales contagion.
- Usability of capital and liquidity buffers during stress is key to avoid large fire sales losses.

## Extension: Strategic sales

- What happens when banks take into account other banks' behaviour?
- This gives rise to a game with strategic substitutabilities: a bank is less likely to sell an asset that other banks are selling.
- Empirically this does not generate big changes in fire-sales losses, relative to the myopic case.
- As expected given that cross holdings of illiquid assets and second rounds losses are small.

Thank you

## Regulatory constraints when deleveraging

Capital ratio:

$$CAP_i(A, E; S) := \frac{E_i - M_i^\top D^{-1} S_i}{\rho_M^\top [(M_i - S_i) \circ (\mathbf{1} - D^{-1} S_i)] + \rho_O^\top O_i},$$

Leverage ratio:

$$LEV_i(A, C, E; S, R) := \frac{E_i - M_i^\top D^{-1} S_i}{(M_i - S_i)^\top (\mathbf{1} - D^{-1} S_i) + C_i + S_i^\top (\mathbf{1} - D^{-1} S_i) - \mathbf{1}^\top R_i + \mathbf{1}^\top O_i}$$

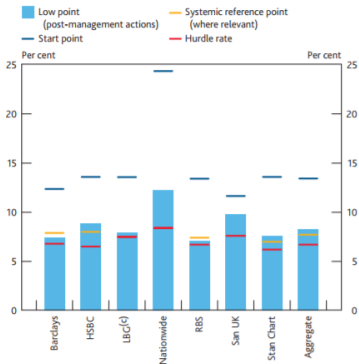
Liquidity Coverage ratio:

$$LCR_i(A, C, L; S, R) := \frac{\lambda^\top [(M_i - S_i) \circ (\mathbf{1} - D^{-1} S_i)] + C_i + S_i^\top (\mathbf{1} - D^{-1} S_i) - \mathbf{1}^\top R_i}{\omega_{out}^\top (L_i - R_i) - \omega_{in}^\top [(M_i - S_i) \circ (\mathbf{1} - D^{-1} S_i)]}.$$

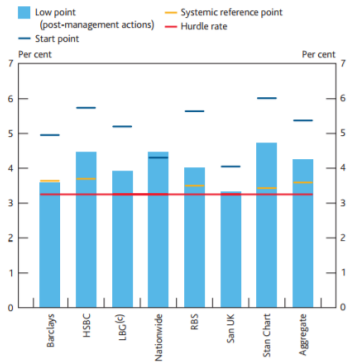
## Marketable asset categories and regulatory weights

Asset	Exposure	LCR haircut	Risk weight
	Govts and CBs	0	0
		15	20
		0	0
		7	0
		15	20
Bonds	Financials	25	35
		30	35
		35	35
		50	50
		100	100
	Non financials	100	100
		50	100
Equities		100	250

# Solvency shock from the 2017 Bank of England stress test



Projected CET1 capital ratios in the stress scenario



Projected Tier 1 leverage ratios in the stress scenario

## Solvency shocks in variants of the 2017 stress test

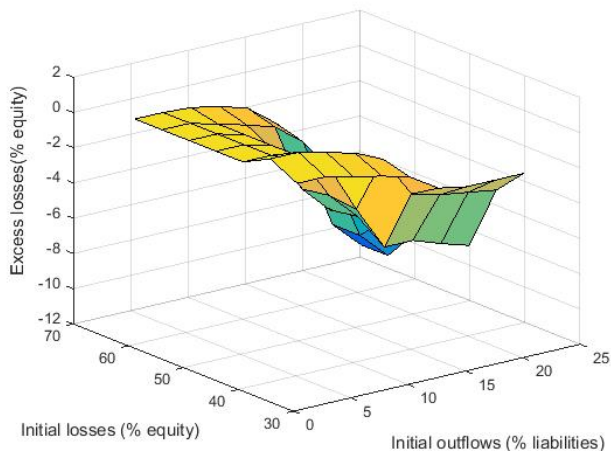
Scenario	Losses (£bn)	Losses (% CET1)
ACS	61	25
+20%	73	30
+40%	85	34
+60%	98	39
+80%	110	44
+100%	122	49



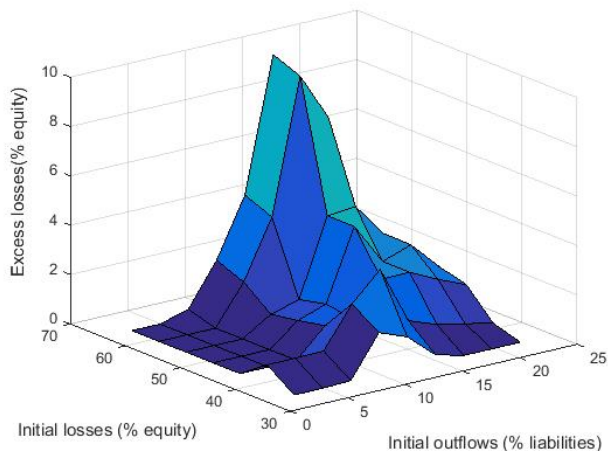
## Funding shocks in variants of the LCR scenario

Scenario	Outflows (£bn)	Outflows (% balance sheet)
-60%	258	5
-40%	387	7
-20%	516	10
LCR	645	12
+20%	774	15
+40%	903	17
+60%	1032	20
+80%	1160	22

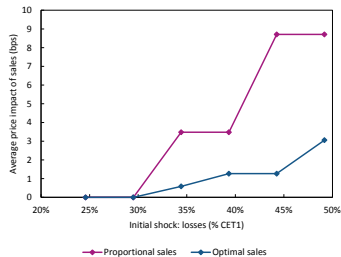
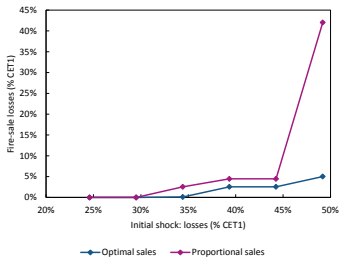
## Losses for composite solvency and funding shocks in excess of the sum of the individual shocks



## Losses for composite solvency and funding shocks in excess of the largest of the individual shocks

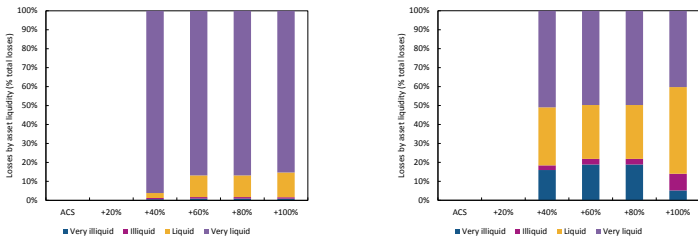


# Comparison with proportional deleveraging



**Figure 1:** Losses (left) and average price impact of sales (right) for solvency shocks under proportional and optimal deleveraging.

## Comparison with proportional deleveraging



**Figure 2:** Losses by quartile of asset liquidity for optimal (left) and proportional (right) asset sales following solvency shocks

## Social planner

$$\min_{\mathbf{S}_A, \mathbf{R}_A} (M_A - \mathbf{S}_A)^\top \delta^{-1} \mathbf{S}_A + \alpha \mathbf{S}_A^\top \delta^{-1} \mathbf{S}_A = \min_{\mathbf{S}_A, \mathbf{R}_A} M_A^\top \delta^{-1} \mathbf{S}_A,$$

subject to the constraints

$$\begin{aligned} CAP(A, E; \mathbf{S}_A) &\geq REG_{CAP} \\ LEV(A, C, E; \mathbf{S}_A, \mathbf{R}_A) &\geq REG_{LEV} \\ LCR(A, C, L; \mathbf{S}_A, \mathbf{R}_A) &\geq REG_{LCR} \\ CASH(A, C; \mathbf{S}_A, \mathbf{R}_A) &\geq 0. \end{aligned}$$

where  $M_A = \sum_{i=1}^N M_i$ ,  $S_A = \sum_{i=1}^N S_i$  and  $R_A = \sum_{i=1}^N R_i$ .

## Social planner

The social planner optimal solution is the same as for individual banks, but accounting for **aggregate** assets holdings rather than *individual* holdings.

Individual banks sales can be socially sub-optimal because of

- How they rank their assets
- How many assets they sell

e.g. if they have small holdings of an asset which is held in large quantities by other banks.

# Counterfactuals

Bank homogeneity vs heterogeneity: Banks becoming more similar can reduce fire-sales losses.

Too big to fail vs Too many to fail: Consolidation between banks can reduce fire-sales losses.





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