How severe are the EBA macroeconomic scenarios for the Italian Economy? A joint probability approach

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December 3, 2019

Abstract

Measures of the severity of macroeconomic scenarios have been widely used in the literature, but a consistent methodology for their calculation has not been developed yet. Against this background, we provide a general method for calculating the joint probability of observing a macroeconomic scenario, which can be applied to a wide variety of structural models. By doing so, we can attach probabilities to scenarios produced with multidimensional economic models to compare their severity and plausibility. We apply our methodology to the 2016 and 2018 EBA stress test scenarios and we provide also reverse stress tests applications. Our results show that for the Italian economy both the 2016 and 2018 EBA scenarios are unlikely, especially the 2016 one. The reverse stress tests allow us to identify the key variables that affect our probabilities.

JEL classification: C30, E30, E44, G10, G20, G28

Keywords: multiple simultaneous equation models; stress tests; financial instability; macroprudential; joint probability.

*We thank Loftur Hreinsson, Elizabeth Jane Casabianca, Pietro Rossi, Lorenzo Forni and Stefania Tomasini for their useful comments.

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1 Introduction

Stress tests are procedures for assessing the potential impact on banks' balance sheet of certain risks, typically financial and macroeconomic shocks. The assessment of such risks is a complex task which requires analyzing the interactions between the financial system and the real economy by means of macro-financial models in order to identify latent fragilities. These types of tools can be usually used also to perform reverse stress test analysis. Reverse stress tests, using the hierarchical structure of the models employed, are procedures that try to quantify how severe a scenario needs to be to bring a given target variable (i.e. banks' capital ratio) below or above a given threshold (for example, a 6% Core Tier 1 ratio threshold). The severity of the scenario is usually assessed looking at the marginal probability distribution of one variable (for example, GDP) obtained by model simulations or by calculating an aggregate index of a set of variables (loss functions). This approach is limited, as it is not able to capture the complexity of the macroeconomic channels affecting the banking system.

In order to create sufficient hypothetical stress on financial institutions, scenarios should be severe enough, but at the same time realistic. In other words, when designing stress tests scenarios, an adequate level of severity must be ensured (i.e. with a significant impact on banks) but not an unlikely one (i.e. non-compatible with the economic structure as well as the history of a given country). Selecting a severe but plausible scenario contributes to increasing the ability of the macro stress tests to become a reliable early warning system. An excessively severe scenario could produce too many false alarms (Henry et al. (2013)). However, the literature still needs to better clarify how to quantitatively assess the severity and the plausibility of a scenario.

Our paper offers a robust and unified framework for the estimation of i) the probability of observing a given scenario and ii) the expected exogenous variables profile for reverse stress testing exercises. One advantage of our approach is that it can be applied to any kind of structural dynamic model and for scenarios jointly defined by a set of variables over time (not just one variable).

Borio et al. (2014) argue that "[...] rather than being part of the solution, stress tests turned out to be part of the problem [...]". In particular, he identifies two main shortcomings. First, the "model" used to simulate financial distress. They are generally large linear autoregressive models with several dimensions and a high degree of complexity, such as the GVAR approach of Olli et al. (2013) [8]. Despite its complexity, the GVAR lacks structural relationships, which prevents the computation of comprehensive probabilities (see Section 2). Moreover, the assumption of linearity represents a limit. Several approaches attempt to avoid the linearity assumption, as Kanas and Philip (2018) [7] who use a non-parametric quantile regression and find that macro variables nonlinearly affect the upper tail distribution (90% and 95% quantiles) of non-performing loans (NPLs). They also find that default probabilities are highly affected by the macro variables selected. The second shortcoming of the existing models is the context in which the stress tests are run.

The theoretical framework underlying macro stress testing allows to assess the impact of macro variables on banks' balance sheet. However, usually the focus is on macroeconomic shocks and how they affect banks' solvency rather than on models that are able to capture the negative feedback loops between banking sector and the rest of the economy.

To overcome the aforementioned limitations, we propose a way to associate a number of macroeconomic variables along a time period (a scenario) to a probability measure (scalar). To this end, we exploit a reduced form of the Prometeia's Quarterly Macro-econometric Model (Tomasini et al. 2018)[6]. The model includes the banking sector and its feedbacks to the other sectors of the economy. Importantly, our approach does not eliminate the structural relationships characterizing the original model.

It is worth noting that our structure works in two directions: from exogenous to endogenous variables to determine scenario probabilities, and from endogenous to exogenous variables to perform reverse stress tests. In this way, we are able to combine the stressed profile of the endogenous variables with the most likely dynamics of the exogenous variables. The intuition of our approach, explained in detail in Section 2, is to track the evolution of the system and to collect all the available empirical moments, such as variances and covariances between variables in their contemporaneous, lagged, and leading versions. The goal is to maintain the structural properties of the model and allow an accurate and consistent estimation of the probability of a scenario, conditional on the joint multivariate distribution of the model's shocks. We apply this methodology to assess the severity of the 2016 and 2018 EBA stress tests macroeconomic scenarios.

As shown in section 3 the joint probability approach delivers the possibility of a robust scenario severity determination by probability comparison. In our application, we obtain that the 2018 and 2016 scenarios have a non zero joint probability mass, respectively 0.50% and 0.15%, suggesting a less severe profile for the 2018 EBA stress test exercise.

Our analysis resemble very closely the approaches found in literature as [3], [4] looking for a formal and general approach for the design of macroeconomic stress testing. With respect to the similar approach we prefer to stress the theoretical content embodied into a structural macroeconomic model. We will see in the following of the paper that this aspect will play a central role for the probability determination.

We focus on the severity of the macroeconomic scenarios and we do not discuss the specific effects of the macroeconomic shocks on the banking sector. Barbieri et al. (2019) [1], using a Large Bayesian VAR model, which accounts for potential spillover between the macroeconomy and the banking sector, propose a model-based approach to assess Italian banks' resiliency to adverse scenarios. The paper is organized as follows. Section 2 discusses the methodology, while Section 3 discusses the empirical application. Section 4 concludes.

2 The methodology

We begin by discussing the standard econometric techniques used in stress testing exercises. If the models used to analyse alternative scenarios is a time series model (VAR and its modifications), there is a limit to the number of observations and time periods that the model can include to yield a non-zero probability. The increase in the number of dimensions (N) and time periods (T) of the model leads the joint probability for the random vector $Y_t \in R^N$, representing the endogenous vector of the simulated model, to converge to zero very quickly. The reason is the (un)desidered Markov property of the model, according to which the joint probability of the simulated scenario, represented by the set of random vectors $Y_1, Y_2, \cdots, Y_t$, is given by the product of the conditional probabilities:

$$P(Y) = P(Y_1, Y_2, ..., Y_t) = P(Y_t|Y_{t-1}) \cdots P(Y_2|Y_1) \cdots P(Y_1) \tag{1}$$

Alternatively, we could use a very simple statistical and structural model in order to exploit the non-markovianity property of the data generation process. The ingredients to build such a model are:

1. Use a structural model that provides a baseline and alternative stressed scenarios. In the following, we discuss the case of a linear model, but the methodology could be extended to nonlinear models as well.
2. Assume the joint distribution of the residuals. We use a multivariate Gaussian distribution as a standard hypothesis.
3. Perform a set of deterministic cumulative shocks on the exogenous variable and then collect cumulative multipliers given the effect of the shocks on each endogenous variable.
4. Compute covariance matrices for the system for each time period of simulation and, given the multiplier matrices system (see below), determine the covariance matrix for the whole scenario.
5. Identify a stress scenario for each endogenous variable of the system.
6. Define the tail of the marginal distribution of the endogenous variable representing the preferences of the macro-prudential policy maker.
7. Evaluate the probability of the scenario as the multivariate joint distribution of the endogenous variable scenario, given the tail specifications (step 6) and the covariance matrix (step 4).

Formally, let us define the deviation between the shocked scenario Y_s and the baseline scenario Y_0 as y_{st}. The same applies for the exogenous variable z_{st}. The endogenous variable $y_{it}, i = 1, \cdots, n$ and exogenous variable $z_{jt}, j = 1, \cdots, p$ are stacked into $Y_t \in R^N, Z_t \in R^P$, respectively. The multiplier at time t is $m_{ijt} = \frac{y_{it}}{z_{jt}}$ and $M_t \in R^{nxp}$, for $t = 1, \cdots, T$ is the respective vector. We build the compact model by collecting each M_t in a matrix as follows:

$$\boldsymbol{M} = \begin{bmatrix} M_t & 0 & \cdots & 0 \\ M_{t+1} & M_t & \cdots & 0 \\ \vdots & \ddots & \cdots & 0 \\ M_T & M_{T-1} & \cdots & M_t \end{bmatrix}.$$

Similarly, for the endogenous, exogenous and shock variables we have:

$$\boldsymbol{Y} = \begin{bmatrix} Y_t \\ Y_{t+1} \\ \vdots \\ Y_T \end{bmatrix}, \boldsymbol{Z} = \begin{bmatrix} Z_t \\ Z_{t+1} \\ \vdots \\ Z_T \end{bmatrix}, \boldsymbol{E} = \begin{bmatrix} E_t \\ E_{t+1} \\ \vdots \\ E_T \end{bmatrix}. \tag{2}$$

Then, we can build the compact system of equations[1] as follows:

$$\boldsymbol{Y} = \boldsymbol{M}\boldsymbol{Z} + \boldsymbol{E}. \tag{6}$$

Assuming first and second moments of the errors and the exogenous variables ($\sigma^2_{e_i}, \sigma_{e_i e_j}, \sigma^2_{z_i}, \sigma_{z_i z_j}$) and their Gaussian multivariate distributions ($E_t \sim \mathcal{N}(0, \Sigma_{E,t})$, $Z_t \sim \mathcal{N}(0, \Sigma_{Z,t})$), we obtain:

$$\boldsymbol{\Xi} = \begin{bmatrix} \Xi_t & \Xi_{t,t+1} & \cdots & \Xi_{t,T} \\ \Xi_{t+1,t} & \Xi_{t+1,t+1} & \cdots & \Xi_{t+1,T} \\ \vdots & \ddots & \cdots & \vdots \\ \Xi_{T,t} & \Xi_{T,t+1} & \cdots & \Xi_T \end{bmatrix} \tag{7}$$

$$\Xi_t = \begin{bmatrix} \Sigma_{Z_t} & \Sigma_{Z_t,E_t} \\ \Sigma_{Z_t,E_t} & \Sigma_{E_t} \end{bmatrix}, \Sigma_Z = \begin{bmatrix} \sigma^2_{z_1} & \cdots & \sigma_{z_1 e_n} \\ \vdots & \ddots & \vdots \\ \sigma_{z_n z_1} & \cdots & \sigma^2_{z_n} \end{bmatrix}, \Sigma_E = \begin{bmatrix} \sigma^2_{e_1} & \cdots & \sigma_{e_1 e_n} \\ \vdots & \ddots & \vdots \\ \sigma_{e_n e_1} & \cdots & \sigma^2_{e_n} \end{bmatrix}. \tag{8}$$

With system (6) and covariance matrices (7) and (8) we arrive at the final multivariate joint distributions:

$$\boldsymbol{Y} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{M}'\boldsymbol{\Xi}\boldsymbol{M}). \tag{9}$$

Model 9 is able to replicate the original macroeconometric model and represents the general solution of the model as represented by a Montecarlo analysis. With model (9) we can measure the probability of the scenario $\boldsymbol{Y}_k$ as

[1] We can see the resulting system as the simultaneous equations format (Canova 2007) of a dynamic simultaneous system. In fact we can write a general macroeconomtric model as a recursive system of simultanoues equations:

$$\mathcal{A}_0 \mathcal{Y}_{t+1} = \mathcal{A}_1 \mathcal{Y}_t + \mathcal{B}\mathcal{Z}_t + \mathcal{E}_t. \tag{3}$$

where $\mathcal{A}_0$ includes simultanoues relationships among system variables. After simple algebra devoted to find the reduced form for the deviation from baseline representation of the above system one get :

$$\mathcal{Y}_T = \Gamma^T_y \mathcal{Y}_t + \sum_{i=0}^{T} \Gamma^{i-1}_y [\Gamma_Z \mathcal{Z}_{t+i} + \Gamma_E \mathcal{E}_{t+i}]. \tag{4}$$

where $\Gamma_y = \mathcal{A}_0^{-1}\mathcal{A}_1$, $\Gamma_Z = \mathcal{A}_0^{-1}\mathcal{B}$ and $\Gamma_E = \mathcal{A}_0^{-1}$. We will record the shocks in the matrices M_i where not only impact and delayed effects of exogenous shocks are included but also the dynamic effects included in $\Gamma^T \mathcal{Y}_t$. In this way we can define the multiplier

$$\frac{Y_{t+i}}{Z_t} = M_i. \tag{5}$$

$P(\boldsymbol{Y}_k \in \bar{\boldsymbol{Y}})$. The tail of the multivariate Gaussian distribution is compared with the policy maker's preference set, $\bar{\boldsymbol{Y}}$. For example, we can assume:

$$\bar{\boldsymbol{Y}_a} = \{y_{1t} > a_1, y_{2t} > a_2\}$$

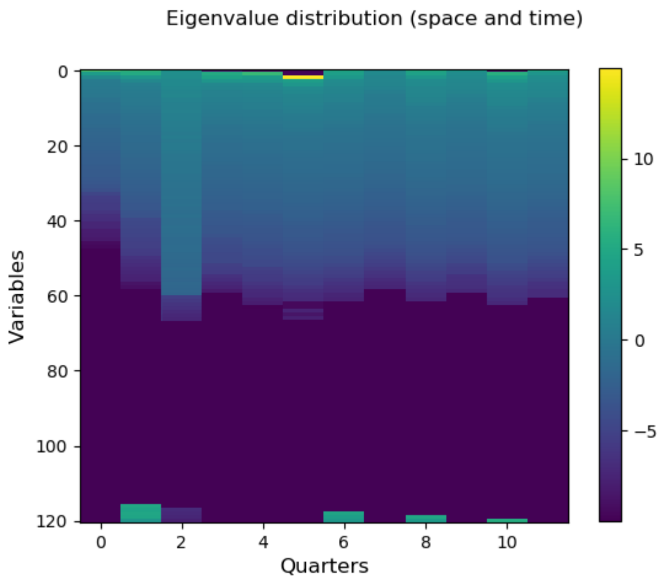
$$\bar{\boldsymbol{Y}_b} = \{y_{1t} < a_1, y_{2t} < a_2\}$$

$$\bar{\boldsymbol{Y}_c} = \{y_{1t} < a_1, y_{2t} > a_2\}$$

i.e. several combinations of the informative set defined as: (i) a right tail (>), (ii) a left tail (<) and (iii) an internal interval (<<) set. In this way, it is possible to estimate a large combination of conditional probabilities to the preferences of the policy maker. For example, in stress tests the probability along the GDP dimension is measured at the left tail ($\infty < Gdp < a$) and that along the unemployment rate at the right tail ($\infty > UR > a$), to then combine them jointly.

The main question is: why does not the probability converge to zero given the high dimension of the probabilistic system? The answer is simple: the system is not Markovian, that is the joint probability is not independent because the model is a structural one, and the whole history of the system is considered (i.e. the entire solution of the dynamic model is acknowledged). By doing so, we do not lose the data required by the structural relationships of the system, keeping the necessary information in the global covariance matrix.

Figure 1: Matrix covariance matrix eigenvalue inspection (Log)



Source: Prometeia calculations

The presence of interdependence is equivalent to the presence in the system of a subset of common factors. By looking at the covariance matrix, we can understand why the probability does not converge to zero. The multivariate Gaussian distribution is:

$$P(x) = \int me^{-1/2x'\Sigma^{-1}x}dx = \int me^{-1/2\sum_i \frac{1}{\lambda_i}e_i^2 y_i}dy_i \tag{10}$$

where the right hand side is obtained after an eigenvalue decomposition of the covariance matrix Σ and where λ_n are the eigenvalues and m is $\frac{1}{\sqrt{(2\pi)^k|\Sigma|}}$. If there are common factors, we should observe large eigenvalues of the covariance matrix that allow the probability to be non-zero. High values of the eigenvalues are a condition for a non-vanishing probability mass. In Figure 3, we show the distribution of eigenvalues by variables and periods. The structure of the model is generated when the first 40-60 variables trigger the rest of the system dynamics and are interrelated. Moreover, we can see that the role of the main components expands over time.

Finally, our framework allows us to determine reverse stress testing analytically. We can solve for the exogenous variables vector $\boldsymbol{Z}$ from system 6 to get:

$$E(\boldsymbol{Z}_k|\boldsymbol{Y}_k) = (\boldsymbol{M}'\boldsymbol{M})^{-1}\boldsymbol{M}'\boldsymbol{Y}_k. \tag{11}$$

System 11 is the expected conditional exogenous profile given $\boldsymbol{Y}_k$, i.e. $E(\boldsymbol{Z}_k|\boldsymbol{Y}_k)$. It allows to identify the most likely path of the exogenous variable. If the original macroeconometric model is well identified, the unicity of the solution for the system 11 is ensured.

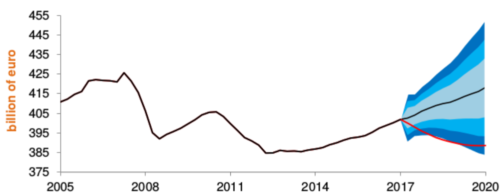
In this section we provide a general model to calculate the probability of a macroeconomic scenario based on the properties of the macroeconometric model. The assumptions are simple but still too general. One of the extensions that is currently planned to be implemented, will be the determination of heterogeneous multipliers depending on the phases of the business cycle in which you can perform stress tests. In section 4 we present the model that allows us to evaluate the probability of scenarios conditioned to the baseline, i.e. to consider an skewed distribution of shocks based on cyclical conditions and to avoid the assumption of homogeneous multipliers. Our preliminary inspection supports the evidence of skewed distributions of exogenous shocks.

3 Applications

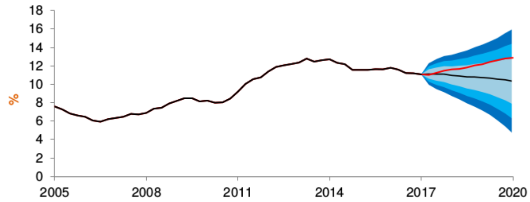
In this section, we show an application of the methodology described in Section 2. For our analysis we rely on Prometeia's Quarterly Macroeconometric Model (Tomasini et al. 2018)[6]. The latter is a large-scale estimated vector error correction model for the Italian economy. It includes several sectors, including firms, households and credit sector at the macroeconomic level. As a practical application of our approach, we analyze a set of endogenous variables (GDP, unemployment rate and the BTP-Bund 10y spread) resulting from the inclusion of EBA shocks in the Prometeia's macroeconometric model as shown in Figure 2. In particular, we show the historical series and the forecasted fan chart resulting from our stochastic simulations.

Then, we compare the fan chart with the 2018 EBA scenario. We can observe a severe fall of GDP and an increase, albeit less severe, in the unemployment rate together with an increase of the BTP-Bund spread. If we calculate marginal probabilities, we get a value of 28.7% for the BTP-Bund spread, 35.2% for the unemployment rate and 6.8% for GDP. The corresponding probabilities for the 2016 EBA scenario the are 22.6%, 30.6% and 3.2%, respectively. However, if we include housing prices, we find a marginal probability of the variable close to zero, suggesting that its profile is implausible.

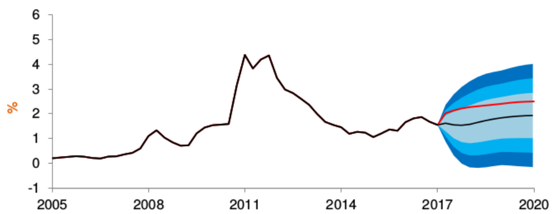
Figure 2: EBA 2018 endogenous variables



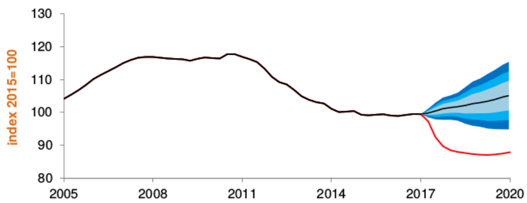
(a) GDP, Probability 3.1% (% dev. from baseline)



(b) Unemployment Rate Probability 30.3% (% dev. from baseline)



(c) SPREAD BTP-Bund, Probability 22.3% (abs dev. from baseline)



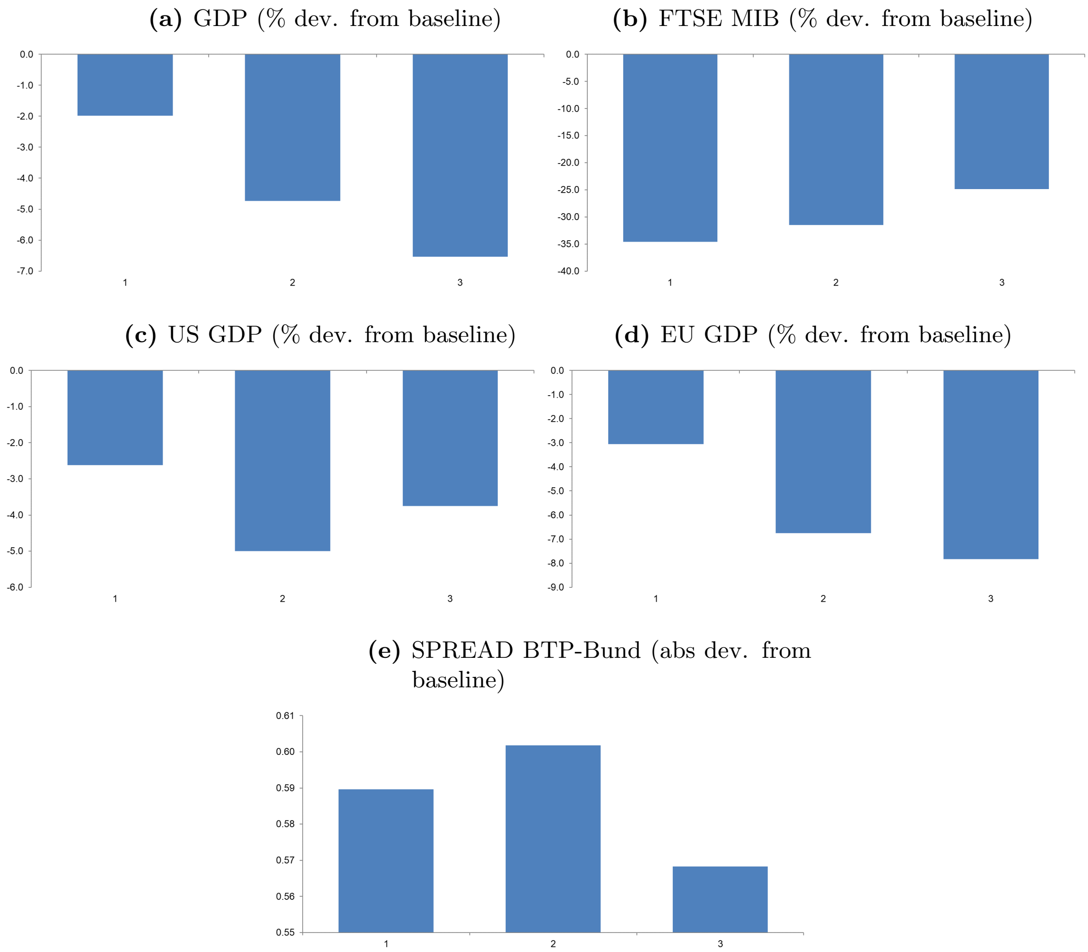
(d) Dwellings Price, Probability 0%: (% dev. from baseline)

Source: Prometeia calculations on EBA data

We use the baseline forecast of Prometeia Associazione and we perform a number of selected shocks on exogenous and estimated errors. Then, we collect the multipliers and the distribution of the estimated errors and exogenous variables. To perform the probability assessment we use the reduced form (6) of the model, thereby preserving the structural relationships of the original model. This property is also achieved by calculating ex-post the dynamic multipliers.

With equation (9) we obtain the joint distribution for the endogenous variables. The dimension of the reduced system is given by $N = 121$ endogenous variables, including the main variables of the macroeconometric model (GDP, unemployment rate, inflation, consumption, aggregate wealth, etc.) and a set of bank interest rates and credit indicators. The number of exogenous variables is $P = 22$, including the exogenous shocks to the main endogenous and exogenous variables. The model is simulated for 12 quarters and therefore the global matrix of multipliers (M) has a dimension of 1452×276, while the covariance matrix $M\Sigma M'$ of 1452×1452.

Figure 3: EBA 2018 endogenous variables



(a) GDP (% dev. from baseline)

(b) FTSE MIB (% dev. from baseline)

(c) US GDP (% dev. from baseline)

(d) EU GDP (% dev. from baseline)

(e) SPREAD BTP-Bund (abs dev. from baseline)

Source: Prometeia calculations on EBA data

In order to calculate the probability of the scenario, we collect data on the exogenous variables included in the EBA scenarios.

As a first step, we apply the probability to both the 2018 and 2016 EBA scenarios. We manipulate the data considering quarterly frequencies and we make a hypothesis on all the variables not explicitly indicated by the EBA, but that are necessary for the simulation of our model.

The profile of the main endogenous and exogenous variables for the 2018 EBA scenario are shown in Figure 3. The adverse shock on the Italian economy is triggered by a negative and persistent shock to the euro area GDP (excluding Italy) (-7.8%), a less severe slowdown of the US economy (-3.8%) and a deterioration of asset prices (initially -30% but slowly recovering).

As for the whole set of endogenous variables, we initially set a preference vector for all variables on the left tail. The probability is 0.201% and 0.069% for the 2018 and 2016 EBA scenario, respectively, indicating that the latter is the most severe from a quantitative point of view.

In order to get a more precise probability for the scenarios, we select a subset of variables as indicated in Figure 4. In particular, we consider a vector with different preferences. We have an adverse preference on the left tail for oil price, exchange rates and GDP. For the unemployment rate, the BTP-Bund 10y spread we have adverse preference on the right tail. In this case, the 2018 EBA scenario yields a probability of 0.503%, while the 2016 EBA scenario a probability of 0.148%. Although the 2016 EBA scenario is still the most severe, the difference between the two scenarios is small.

Figure 4: Scenario preference configuration

Variable	Tail
Oil price	rhs
Exhange Rate €/$	rhs
GER Bund Rate	rhs
Ermergin Gdp	lhs
US Gdp	lhs
Euro Area Gdp	lhs
FTSE MIB	lhs
Italy GDP	lhs
Unemployment rate	rhs
Spread Btp-Bund 10y rhs	rhs
Euribor 3M	rhs

Source: Prometeia calculations

We can also detect which variable causes a decrease in probabilities. When we include the price of dwellings (Figure 2d) we obtain a joint probability of 0%, as expected from preliminary inspection of marginal distribution. Therefore, dwelling prices are the most implausible endogenous variables, as it is assumed to persistently deviate from its baseline value to -20%. From an empirical point of view, Italy has not experienced such shock in a 3 year time period. To better appreciate the plausibility of the scenario, we perform a second application: the reverse stress test.

We use the 2018 EBA endogenous set of variables (Figure 3) and fit it to our model specification. We use system (11) to obtain the exogenous profile with the higher conditional expected value consistent with our model. The adverse additional shock profile of the exogenous variables are shown in Figure (5). They consist in an initial adverse shock to the oil price, the euro area GDP and the US economy. Unemployment rate starts growing, while the ECB increases its policy rate. The BTP-Bund spread increases over the entire period. It is worth noting that neither for GDP nor for equity prices an additional shock is required. Overall, these results allow us to quantify the additional shocks needed to replicate the EBA scenario given the Prometeia macroeconometric model. They also help to identify how the EBA scenario can be improved to make it more plausible.

Figure 5: Distribution of the sub-periods by quartile of selected macroeconomic indicators % dev from baseline

Source: Prometeia Calculations based on EBA data

4 Extensions: Conditioning probability to the business cycle phase

The method presented in the previous section assume a constant structure of the joint probability distribution. In some sense, it is unsatisfactory as it provides the same probability measure for a stress test scenario. It would be preferable to perform stress testing exercises been conditioned to the actual macroeconomic condition, with a more flexible model taking into account the current macroeconomic conditions. A stress test implemented during a recession could have a probability measure much higher than performed during expansionary phases. We could consider several ways to complicate such model and in this section we show the simplest extension: a two state multivariate distribution. The goal is to measure the joint probability of a macroeconomic scenario considering the positive and negative phases of the business cycle.

As the simplest example for an extension of model in section 2 we provide the following two-states system:

$$\boldsymbol{Y} = \begin{cases} \boldsymbol{M}^{-}\boldsymbol{Z} + \boldsymbol{E}^{-} & \text{with probability } p \text{ (negative output gap)} \\ \boldsymbol{M}^{+}\boldsymbol{Z} + \boldsymbol{E}^{+} & \text{with probability } (1-p) \text{ (positive output gap)} \end{cases}$$

This model allows to perform conditional stress-tests into two possible states depending on the phase of the business cycle. If the output-gap is negative (positive) we can assume a skewed distribution of shocks. The same hypothesis holds for the several endogenous variables with the heavier tail, depending on the particular variable of the model. At the same time we could expect a non-zero multiplier for exogenous shocks impacting during recessionary business cycle phases. Formally, it translates into $E(\boldsymbol{M}^{-}\boldsymbol{Z}) = \boldsymbol{\mu}_{z-} > 0$, $E(\boldsymbol{E})^{-} = \boldsymbol{\mu}_{e-} > 0$ and a log normal distribution for $Y^{-} \sim log(\mathcal{N})(\boldsymbol{\mu}_{z-} + \boldsymbol{\mu}_{e-}, \Sigma_{E,t})$. Figure 6 depicts the whole picture of the model for positive and negative phases and the use of the prior distribution for the output-gap, weighting the two multiplier sub-systems.

As a first numerical example, we consider the hypothesis in which the conditional probability is obtained by keeping the estimated parameters of t he model and the matrix of the multipliers, $\boldsymbol{M}^{-} = \boldsymbol{M}^{+} = \boldsymbol{M}$, constant between the two states.

What we assume is the heterogeneity of the shocks obtained bootstrapping the sample distribution of shocks in the two states, conditioned to the positive or negative output-gap, i.e, considering the distributional moments of E^{+} and E^{-}.

Figure 6: Conditional Probability distribution,

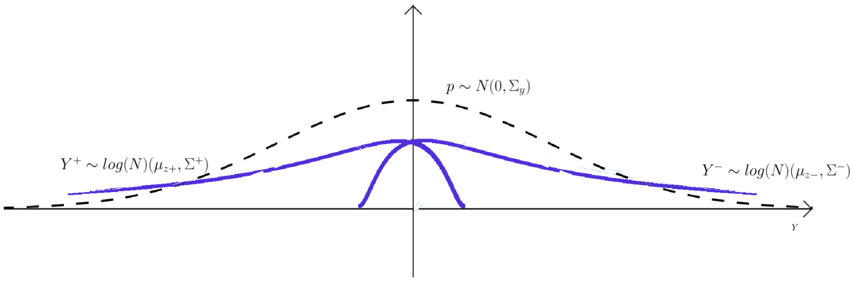
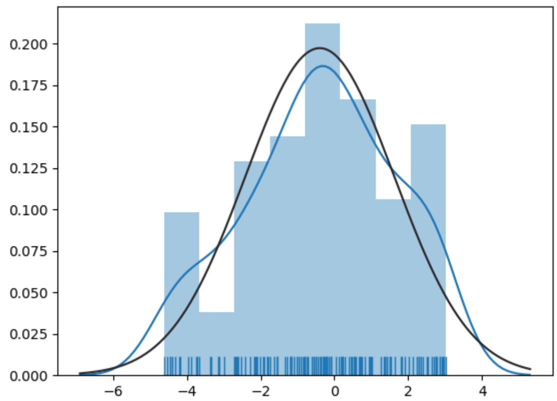


Figure 7: Output Gap distribution, Empirical (bars), Kernel (blue) and Gaussian (black)



Source: Prometeia calculations

First of all, we estimate the kernel distribution as in figure 7 in order to obtain the probability of a positive state of the macroeconomic cycle, namely a positive output gap for the Italian economy. Then, we can calculate the conditional probability of a particular scenario given the probabilities of the negative and positive state, $P(Y^+)$ and $P(Y^-)$.

We can observe in the figure 8 that the simulated endogenous distributions has a skewed shape and that it approximates a lognormal bivariate pdf. On top of this distribution, we can calculate a transformation by shifting and logging the original empirical distribution considered for each variable and time period[2].

In this way, we can lead back to a normal multivariate distribution specific for each of the two states.

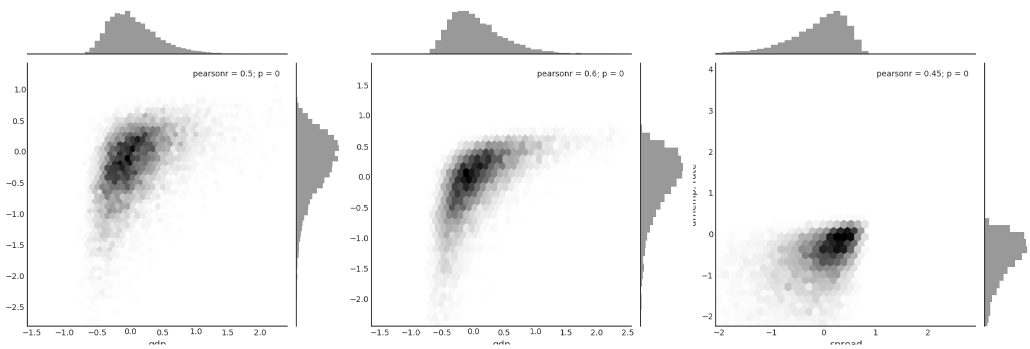
Once we have obtained our normal multivariate distribution, we can apply the same procedure presented in the previous sections. That is, we apply the EBA 2018 scenario as in section 2 for each to the two states. In this way we obtain a scenario probability in the positive output-gap state of 0.003 per cent and for the negative state of 8.4. This means that the 2018 EBA scenario is much more likely during recessionary state of the economy, as expected.

[2]We fit a shifted log normal distribution per each variable belonging to the state of the economy, ie. $y_{it}^S \sim log\mathcal{N}(0, \sigma_{S,i,t}^2, \delta_{S,i,t}), S = +, -, i \in N, t \in T$, where $\delta_{S,i,t}$ is the location parameter.

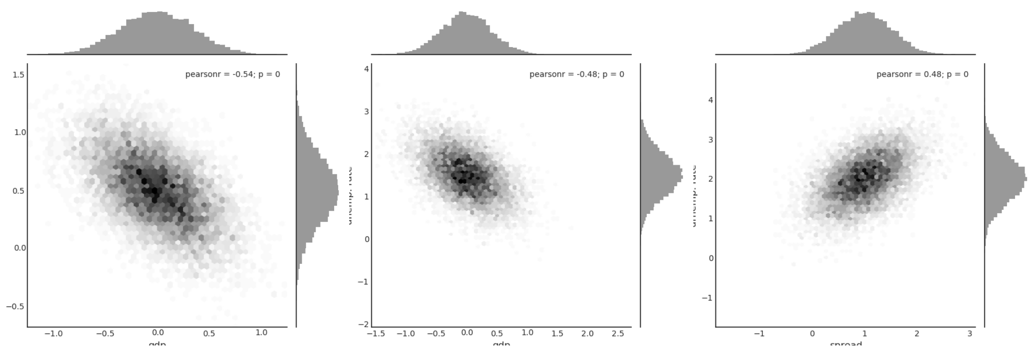
Finally, applying the conditional probability calculation recursively during the sampling period, we can obtain a dynamic probability of 2018 scenario conditioned to the output-gap, as shown in figure 9. As expected, at the begin of the Great Recession the likelihood of the scenario has designed for the 2018 stress-test exercise has increased. Recently, the probability value is lowering as the macroeconomic conditions for the italian economy has improved in terms of the output gap measure.

Figure 8: Pair of bivariate simulated distributions. Positive state. Original (1st row) and transformed (2nd row) data. Transformation : $log(X) + \delta$

(a) GDP vs SPREAD **(b)** GDP vs URD **(c)** SPREAD vs URD

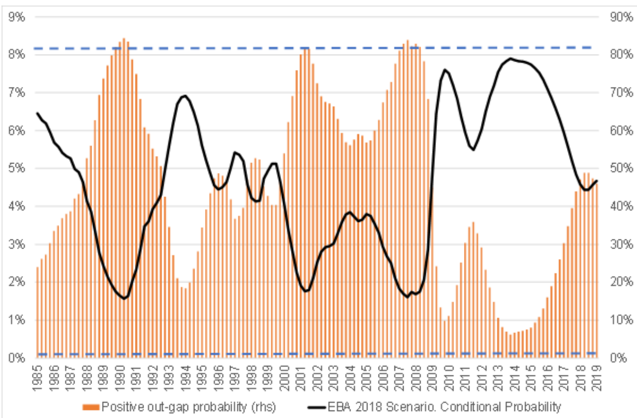


(d) GDP vs SPREAD **(e)** GDP vs URD **(f)** SPREAD vs URD



Source: Prometeia calculations

Figure 9: 2018 EBA Scenario, conditional probabilities over time



Source: Prometeia calculations

5 Conclusions

This article looks at the severity of the EBA shocks with respect to the macroeconomic scenario considered in past stress tests for Italian banks. To the best of our knowledge, no standard analytical method exists in the literature to measure the plausibility and severity of stress testing scenarios. We provide a very simple analytical model, which allows a precise computation of the probability of the scenario.

This approach, if performed together with reverse stress tests, is suitable for the detection of exogenous variables that are not plausible. In this framework, the macroeconomic scenario designed by the EBA in 2016 is more severe than the 2018 one. Compared to the latter, our methodology allows us to obtain an additional profile of exogenous variables from Prometeia's macroeconometric model, consistent with the profile of endogenous variables of the EBA. In this context, a much severe shock on the BTP-Bund spread, unemployment rate and the euro area GDP is necessary to induce the EBA scenario, indicating that the latter potentially includes implausible variables.

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