# $BSLoss$ – a comprehensive measure for interconnectedness

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**Abstract**

We propose a measure for interconnectedness: $BSLoss$, the banking system loss which describes the increase in expected credit losses due to contagion risk. For this purpose, we construct an algorithm to model the transmission of credit risk in the interbank market via a multiple-round-process. Core to the algorithm is the simulation of how a deterioration of credit quality of debtor banks spills over to creditor banks via the interbank credit channel. The transmission of an exogenous shock to one or a group of banks is modelled on the basis of the empirical relationship between the Tier 1 capital ratio and the probability of default of banks. As a consequence of the contagion mechanism a devaluation of banks' assets and an increase in credit losses can be observed. Compared to other measures for interconnectedness $BSLoss$ comprises several merits: It is easy to interpret in economic terms, it reacts sensitively even to small changes in credit risk, and it accounts for the recursive nature of the banking network. The algorithm may be applied to analyze the effectiveness of systemic capital buffers to curb contagion risk in the banking system.

## 1 Introduction

Credits between banks have a two-sided effect on financial stability. On the one hand, interconnected banks may improve risk sharing and diversification, thereby alleviating their risk position to idiosyncratic shocks, as noted by Allen and Gale (2000), and Freixas, Parigri, and Rochet (2000). On the other hand, the network exposes all banks to the risk of contagion, that is, an adverse shock to one bank or a group of banks can spread to other interconnected banks, which results in distress, or - in the worst case - in a default of these banks. In this spirit, BCBS (2013) classifies the level of interconnectedness as one main driver for systemic risk in the banking system.[1] In the light of the recent financial crisis the risk of contagion has increasingly become a matter of importance to regulators. Therefore, this paper focuses on the adverse externalities interbank credits may have on the stability of the banking system.

When analyzing interconnectedness, it is essential to identify an economically meaningful unit to measure its costs. We propose a model to measure interconnectedness by the increase in expected credit losses of the banking system caused by contagion through interbank credits. The work builds

[1] Systemic risk, defined by BIS, IMF, and OECD (2001), is a risk, that an event will trigger a loss of confidence in a substantial portion of the financial system that is serious enough to have adverse consequences for the real economy.

on the DebtRank introduced by Battiston, Puliga, Kaushik, Tasca, and Caldarelli (2012). Our model allows to monitor the build-up of vulnerabilities over time and reason ex-ante macroprudential actions. The effect of different types of shocks and different policy instruments can be analyzed consistently. The model is flexible so that other financial institutions apart from banks (eg hedge funds, insurers etc.) could also be included in the study.

The paper contributes in two ways – one is of a descriptive nature and the other of a normative nature. First, our model enhances the DebtRank, both in terms of the theoretical and empirical foundation. In our model the credit loss is derived from well-established theoretical credit risk measures. It reflects the balance sheet loss of the banking system due to a credit deterioration of banks under a stressed scenario. In the stressed scenario we simulate an increase in the probability of default of banks, given a shock has hit one bank or a group of banks and is transmitted via the channel of interbank credits. The simulation process is empirically founded by a multivariate logit-regression which explains the increase in the probability of default of a bank by a deterioration in its regulatory variables (eg Tier 1 capital ratio). Second, the paper contributes to a better understanding of the effectiveness of different policy actions to different types of shocks. In this respect, simulations with alternative additional capital buffers imposed on banks and different types of shocks from inside or outside the banking system (eg burst of a housing bubble) are compared to each other and conclusions are drawn.

The paper proceeds as follows. Section 2 gives an overview on the relevant literature with the focus on the DebtRank. The benefits of our model as compared to the DebtRank are set out. In section 3 we describe how contagion spreads across an interbank credit network and develop a suitable algorithm to capture the phenomenon. Section 4 presents the results of two policy experiments. In section 5 we discuss policy implications in the context of the ongoing debate on regulatory reform and conclude.

## 2 Literature

Interconnectedness in the banking system may be characterized by different types of contagion channels. For example, Jobst (2012) provide an interesting analysis by focusing on the liquidity channel. He combines option pricing with market information and balance sheet data to generate a stochastic measure of the frequency and severity of a system wide liquidity event. Upper (2010) provides a comprehensive overview of different simulation models which study different types of contagion channels on the liability and the asset side, such as common liquidity pools, interbank lending and payment systems. In our analysis, contagion is concerned with the interbank credit channel and how a deterioration in credit quality of one bank or a group of banks spreads in the banking system and adversely affects the credit quality of other interconnected banks.

Most of the existing studies about the adverse effects of interbank credits on the stability of the banking system can be attributed to two strands of literature. The first one refers to default cascade models. The intuition behind this approach is clear: A bank default triggers a loss on interbank credits for its creditor banks. This, in turn, may trigger a default of the creditor banks and a corresponding loss for the creditor' creditor banks, and so forth. In this spirit, Eisenberg and Noe (2001) propose a static model which is characterized by a clearing payment vector. This vector represents a function of the operating cash flows of the members of the financial network and satisfies the requirements of limited liability, debt priority and pro-rata reimbursements. Rogers and Veraart (2012) extend the modelling framework of Eisenberg and Noe (2001) by introducing default costs in the system. They analyze situations in which solvent banks have the incentives to rescue failing banks and conclude how such a rescue consortium might be constructed. Focusing more on empirical findings, Mistrulli (2011) explores how banks' defaults propagate within the Italian interbank market by using a unique data set including actual bilateral exposures. He finds that contagion based on actual exposure patterns tends to exceed contagion based on hypothetical exposure patterns (eg entropy maximization method) which previous works often had to rely on due to the lack of actual bilateral exposure information. Memmel and Sachs (2011) develop a default cascade model with stochastic loss given defaults (LGDs) which follow approximately the U-shaped Beta distribution and is calibrated on realized recovery rates from defaulted interbank exposure. They conclude that contagion in the German interbank market can occur and that the number of bank defaults increases on average if a stochastic LGD is assumed instead of a constant one.

The second strand of literature refers to centrality measures, which aim at identifying the most important node in a network. Different centrality measures exist which reflect different interpretations of importance. Landherr, Friedl, and Heidemann (2010) provide a critical review of different centrality measures. One of the most simplistic measures is the degree centrality which takes into account the connection one node has to other nodes. More complex measures are recursive centrality measures. According to this concept, the centrality of one node in the network does not only depend on the degree of its nodes, but also on the centrality of those nodes it is connected with. This means, the centrality (or level of interconnectedness) of one node spreads across the network and influences the degree of centrality of the direct and indirect neighboring nodes. As a result, the centralities of all (connected) nodes influence each other recursively. Recursive centrality can also be described as a weighted sum of all direct and indirect connections of any length. This concept is formalized mathematically in the standard eigenvector centrality by Bonacich (1972). Another version of recursive centrality measures is the PageRank, developed by Google's co-founder Larry Page, to assess the importance of websites. In the wake of the recent financial crisis the concept of recursive centrality measures has gained popularity as an indicator for interconnectedness in the banking system. To name a few, ECB (2012) and Brunnermeier, Clerc, and Scheicher (2013) applied recursive centrality measures to assess structural vulnerabilities and the level of interconnection of banks. Martinez-Jaramillo, Alexandrova-Kabadjova, Bravo-Benitez, and Solorzano-Margain (2014) propose a unified measure of centrality. Applying the Principal Component Analysis Martinez-Jaramillo et al. (2014) suggest a unique index of centrality which incorporates information of several centrality measures.

Default cascade models and centrality measures have their merits and limitations, as outlined by Battiston et al. (2012). Default cascade models provide an easy economic interpretation of contagion. Their measure, ie the loss occurring in the banking system consequently to a bank's default, enables to draw comparisons between different banking systems and with one banking system at various points in time. However, default cascade models are typically restricted to an ex-post view, because they assume that only a default of a bank, ie a materialized loss, can result in adverse spill-over effects. This ignores the fact that a relatively small credit deterioration of a bank could already have negative consequences for the creditor banks' solvency, because their portfolio is exposed to a higher expected loss from an ex-ante perspective. In this respect, default cascade models can be characterized as backward-looking and less responsive to risk.

By contrast, centrality measures can be described as forward-looking and more responsive to risk; vulnerabilities due to an increased level of interconnectedness can be captured before losses have materialized. However, centrality measures are hard to interpret in economic terms as opposed to other forward-looking measures, such as the expected loss which is expressible in monetary units. Thus, quantifying adverse effects from interconnectedness and reason macroprudential actions to mitigate these effects are difficult to achieve based on centrality measures. Finally, they are not suitable to compare between different banking systems and for one banking system at various points in time.

Against this background, Battiston et al. (2012) develop the DebtRank, which combines the benefits of recursive centrality measures and default cascade models to overcome the above mentioned limitations. The DebtRank aims at measuring the economic loss caused by contagion after some predefined shock has hit one bank or a group of banks. In essence, the transmission of a shock is mirrored by an increase in the level of distress of the interconnected banks. Battiston et al. (2012) describe the level of distress for banks by a continuous variable ranging between 0 and 1, where the lower boundary means 'undistressed' and the upper boundary means 'default'. They construct an algorithm which postulates how banks' levels of distress depend on each other. Accordingly, the level of distress of a bank, say bank A, is influenced by the level of distress of its debtor banks weighted by the relative exposure. The relative exposure describes a debtor specific ratio and equals the credits between bank A and its debtors over the core capital of bank A. It reflects the relative portion of capital of bank A which would be lost in case of a default of the debtors banks and a recovery value of 0. To tame reverberations it is assumed each bank can propagate its distress only once. In the simulation, each bank starts with an identical initial level of distress of 0. After an exogenous shock has hit one bank or a group of banks - ie their level of distress is set to some amount between greater 0 and equal 1 - the algorithm calculates the new level of distress for all banks. To measure the economic loss the difference between the banks' total assets weighted by the levels of distress after contagion and before contagion is calculated.

The DebtRank provides plenty of interesting insights into the adverse effects of interconnectedness. However, in terms of theoretical interpretation and empirical analysis the DebtRank leaves room for some refinements. First, the level of distress, one key variable of the DebtRank, remains abstract and unobservable. Thus, we propose a model which interprets the banks' level of distress as their level of credit quality, which can be observed by their (historical) probability of default. Second, the contagion and mutual interference process of banks' distress postulated by the DebtRank is intuitive, but lacks an empirical verification that distress just spreads in this way; eg why should the impact of distress between banks be just proportional to the relative exposure? To address this issue, our model derives the contagion effect from the empirically verifiable relationship between a bank's Tier 1 capital ratio and its probability of default. Third, the DebtRank allows only once for reinfection between banks. However, we show that risk propagation, which captures multiple round effects, can be essential to assess the persistence of the adverse effects of contagion accurately.

## 3 Methodology

To simulate the contagion effect we focus on assessing the impact a change in the debtor bank's probability of default (PD) has on the creditor bank's PD. In this respect, we choose a two-step approach: First, we investigate the impact the debtor bank's PD has on the creditor bank's Tier 1 capital ratio – defined as Tier 1 capital over risk-weighted assets (RWA). Second, we analyze the impact bank's Tier 1 capital ratio has on its own PD.

In the first step, we estimate the influence the debtor bank's PD has on the creditor bank's Tier 1 capital ratio following regulatory and accounting requirements. In essence, we aim to mimic banks' risk management practices based on external reporting requirements. An increase in the debtor bank's PD results in a deterioration of the credit quality of the portfolio of its creditor banks because the creditor banks may be exposed to higher expected credit losses and to higher unexpected credit losses. The former one is captured by an asset devaluation on the creditor banks' balance sheet according to the applicable accounting standards, eg in the form of loan loss allowances (LLA) which are deducted from their Tier 1 capital. The latter one is reflected by higher regulatory charges according to the Basel Accords, eg in the form of RWA. Both effects drive down their Tier 1 capital ratios. In the second step, we use a logit-regression to estimate the percentage point change of the PD of a bank given a one percent change of its Tier 1 capital ratio. According to Packer and Tarashev (2011) high quality capital measures, such as the Tier 1 capital ratio, are one important factor to assess bank's credit quality. To isolate the effect the Tier 1 capital ratio has on the PD, a set of control variables are included in the logit-regression, which reflect (but are not limited to) profitability and liquidity ratios.

We then develop an algorithm which iteratively computes the change in each bank's PD after an exogenous shock has hit one bank or a group of banks. Exogenous shocks reflect a sudden deterioration in credit quality of the shocked banks resulting in default or distress. In our algorithm a bank defaults if it is assigned a PD of 1 during the iteration. Additional stress the banks suffer from is expressed through the assignment of PDs which are higher – but still smaller than 1 – compared to the respective PDs which have been assigned in previous iteration steps. Given the relationship between the debtor bank's PD and the creditor bank's PD, default or distress of one bank or a group of banks may result in a subsequent increase of PDs of its creditor banks, and the creditors' creditor banks, and so on. In order to capture this mechanism the algorithm models a multiple round contagion effect where the PD of all the creditor banks deteriorates, which are connected (directly and indirectly) with those debtor banks subject to the exogenous shock. The increase in banks' PDs results in higher expected credit losses, and thus a devaluation of banks' assets, which we propose as a measure for the adverse affects of interconnectedness caused by contagion through interbank credits.

### 3.1 General algorithm

After having explained the conceptual idea of our algorithm in a narrative way we now put it into a mathematical format. Before doing so we need to introduce some notation. Let us denote by $W_{ij}$ the interbank credit from bank $i$ to bank $j$. The variable $P_0(i)$ describes the PD bank $i$ has in the "unstressed" environment at time $t = 0$. Let furthermore $P_t(i|A)$ be the PD of bank $i$ at time $t$ conditional on the exogenous shock event $A$ in $t = 1$. Each $P_t(i|A)$ is a continuous variable with $P_t(i|A) \in [0, 1]$. The statement $P_t(i|A) = 1$ is equivalent to the default of bank $i$ at time $t$.

The starting point in the generation of stressed PDs is a series of capital and leverage ratios, which are updated subsequently for all banks in each step of the iteration. The leverage ratio for bank $i$ at time $t$ is defined by Tier 1 capital divided by total assets (net of LLA), ie $Lev_{i,t} = \frac{Tier1_{i,t}}{TA_{i,t}}$. In contrast, we use the Tier 1 capital ratio $CapRat_{i,t} = \frac{Tier1_{i,t}}{RWA_{i,t}}$ as a risk-based measure to describe the solvency of bank $i$ at time $t$ according to supervisory capital standards. In order to compute the risk-weighted assets $RWA_{i,t}$ we follow the methodology adopted for the Internal Ratings Based (IRB) approach of the Basel 2 / Basel 3 supervisory capital frameworks. To comply with the rules of the Foundation IRB approach for exposure to the banking sector we apply for the loss given default $LGD = 45\%$ and for the residual maturity $M = 2.5$.

First, the algorithm updates in parallel total assets, Tier 1 capital and risk-weighted assets. Using the notation introduced above we obtain the following expression for the changes of these quantities from one period to the next:

$$\begin{aligned} TA_{i,t} &= TA_{i,t-1} - \sum_j W_{ij} \cdot LGD \cdot (P_{t-1}(j|A) - P_{t-2}(j|A)) \\ Tier\ 1_{i,t} &= Tier\ 1_{i,t-1} - \sum_j W_{ij} \cdot LGD \cdot (P_{t-1}(j|A) - P_{t-2}(j|A)) \\ \Delta_t RW_j &= max\{0, RW(P_{t-1}(j|A), LGD, M) - RW(P_{t-2}(j|A), LGD, M)\} \\ RWA_{i,t} &= RWA_{i,t-1} + \sum_j \Delta_t RW_j \cdot W_{ij} \end{aligned}$$

Note, that including the maximum in the formula for the change in risk-weights $\Delta RW$ takes into account some specifics of the IRB risk-weight functions. From a certain threshold value of PD the risk-weights do not necessarily increase anymore given that risk-weighted assets are defined in such a way that they only cover unexpected losses. Further technical details on the computation of $Tier\ 1_{i,t}$, $TA_{i,t}$ and $RWA_{i,t}$ can be found in the Appendix A1.

Second, our algorithm exploits the marginal effect a one percentage point change of bank's Tier 1 capital ratio has on its PD which is given by

$$\beta_{caprat} \cdot (P_{t-1}(i|A) - P_{t-1}(i|A)^2), \tag{1}$$

see Appendix A2 for a step-by-step development of this term. This marginal effect is important for our approach as it is used in our algorithm to compute the change in the default probability of each bank directly from the change in the capital ratio.

We extract the coefficient $\beta_{caprat}$ from the logit-regression

$$Pr(default_t) = F(\alpha + \beta_{caprat} ln(CapRat_{t-1}) + \gamma X_{t-1}),$$

where $F(z) = e^z/(1 + e^z)$ is the cumulative logistic distribution and $Pr(default_t)$ is the probability of the dependent variable equalling a "success" in period $t$. In our case a "success" means a default. In this regression the matrix $X_{t-1}$ stands for the observation of a set of the control variables reflecting the following characteristics of a bank: capital adequacy, asset quality, quality of management, profitability, and liquidity. We use the variables of the year $t - 1$ to explain the defaults in year $t$. More detailed specifications and regression results can be found in Table 3 in Appendix A2.

Remember that the variable $P_0(i)$ describes the PD prior to the event $A$, ie at $t = 0$. Now, denote by $S$ the set of banks subject to the exogenous shock. We assume that the conditional PDs at time $t = 1$ can be expressed by

$$P_1(i|A) = P_0(i) + \varphi \leq 1 \text{ for all } i \in S$$
$$P_1(i|A) = P_0(i) \text{ for all } i \notin S.$$

The variable $\varphi$ represents a positive parameter reflecting the level of stress imposed on $S$, which, in the extreme case, leads to a bank default, ie $P_1(i|A) = 1$.

The algorithm for $t \geq 2$ updates in parallel all the conditional probabilities of default $P_t(i|A)$ taking into account all the conditional probabilities of default $P_{t-1}(i|A)$ as well as Tier 1 capital $Tier\ 1_{i,t-1}$, total assets $TA_{i,t-1}$ and risk-weighted assets $RWA_{i,t-1}$ of the previous round for all banks $i$. A bank is considered as defaulted and its PD is set to 1, if some pre-defined default criteria are met. The default criteria may be reflected by a hard default, such as $Tier1 < 0$. More sensitive default criteria may be reflected by regulatory requirements, or other boundaries which are not set explicitly by the supervisor but do also influence bank's credit quality. In our algorithm, we choose lower boundaries for the Tier 1 capital ratio and leverage ratio as default criteria, which we describe by $CapRat_{crit}$ and $Lev_{crit}$. In principle, the specific values for $CapRat_{crit}$ and $Lev_{crit}$ are to be configurated in accordance with the banking-specific environment the algorithm is applied to. For banks which do not default, the probability of default increases in accordance with the change in the probability of default of its counterparties according to expression (1).

The dynamic stops after a finite number of multiple steps, let us say at $t = T$. Otherwise, bilateral interbank connections would re-infect each other ad infinitum. In this respect, we deviate from the DebtRank where re-infection is allowed only once. In our opinion, third and more round effects are essential to assess the persistence and severity of contagion risks accurately.

The box below includes a formalized version of the algorithm.

**Contagion Algorithm**

$P_1(i|A) = P_0(i) + \varphi \leq 1$ with $\varphi > 0$ for all $i \in S$, and $P_1(i|A) = P_0(i)$ for all $i \notin S$

$t = 2$, iterate

$$P_t(i|A) = \begin{cases} 1 & \text{if } CapRat_{i,t} < CapRat_{crit} \text{ or } Lev_{i,t} < Lev_{crit} \\ min\{1, P_{t-1}(i|A) + \beta_{caprat} \cdot (P_{t-1}(i|A) - P_{t-1}(i|A)^2) \cdot \Delta_t ln(CapRat_i)\} & \text{else,} \end{cases}$$

$t = t + 1$, until $P_T(i|A) - P_{T-1}(i|A) < \epsilon$ with a small value $\epsilon > 0$.

In the algorithm $\Delta_t ln(CapRat_i)$ reflects the percentage change in Tier 1 capital ratio which is given by

$$\Delta_t ln(CapRat_i) = ln(CapRat_{i,t}) - ln(CapRat_{i,t-1}) = ln(\frac{Tier1_{i,t}}{RWA_{i,t}}) - ln(\frac{Tier1_{i,t-1}}{RWA_{i,t-1}}).$$

Some caution is necessary when the PDs are updated in each step of the iteration since the formula included in the box holds only for infinitesimally small changes of $ln(CapRat_i)$. For larger changes of the capital ratio rules from calculus give us the expression

$$P_t(i|A) = \frac{\left(\frac{CapRat_{i,t}}{CapRat_{i,t-1}}\right)^{\beta} \left(\frac{P_{t-1}(i|A)}{P_{t-1}(i|A)-1}\right)}{\left(\frac{CapRat_{i,t}}{CapRat_{i,t-1}}\right)^{\beta} \left(\frac{P_{t-1}(i|A)}{P_{t-1}(i|A)-1}\right) - 1}, \tag{2}$$

see Appendix A3 for a derivation.

We may assume that the algorithm terminates because the sequence $P_0(A)$, $P_1(A)$,..., $P_t(A)$,... of vectors of the $PDs$ has a monotone limit by the following reasons:

- By construction it has the vector $\mathbb{1}$ (each component equals 1) as an upper bound.
- It is monotone increasing because $\beta_{caprat} > 0$, $P_{t-1}(i|A) - P_{t-1}(i|A)^2 \geq 0$ and $CapRat_{i,t+1} < CapRat_{i,t}$.
- It is not continuous from below because banks may fail. They are assigned a $PD$ of 1 in case their Tier 1 capital ratio or leverage ratio fall below $CapRat_{crit}$ or $Lev_{crit}$. However, the iteration can be restarted after each failure. Due to the fact that the number of banks is finite the existence of a monotone limit is ensured.

The algorithm gives us the following result for the considered population of banks:

$$BSLoss = \sum_j (TA_{j1} - TA_{jT}) \tag{3}$$

where $T$ is the numbers of iterations carried out by the contagion algorithm.

The quantity $BSLoss$ measures the balance sheet loss of the banking system induced by the event $A$. Using this approach a ranking of the banks' level of interconnectedness may be derived. $BSLoss$ takes into account the reduction in total assets due to an increase in credit losses between $t = 1$ and $t = T$. In case the event $A$ is the default of a single bank, we may rank the bank's level of interconnectedness in accordance to the banking system loss its default induces to the system. In this regard, we can determine those banks which are too-interconnected-too-fail.

Given that the matrix W describes a very complex network it turns out to be difficult to describe properties of each sequence $PD_t(i|A)$ or for the $BSLoss$ that go beyond the fact that these quantities converge. For this reason we use a simple academic example to develop some intuition for the algorithm and the sequences it generates. The academic example can be found in Appendix A4.

## 3.2 Extensions of the algorithm

Our model offers various implications for macroprudential policy. For example, the model can be used to analyze the level of banking system loss for different types of shocks (eg shocks stemming from outside the banking system). Furthermore, it allows us to compute by how much the banking system loss will be absorbed if we increase the Tier 1 capital for certain banks, in particular for those banks which are considered as highly interconnected. This makes our algorithm a useful tool in determining the size of capital surcharges to curb contagion risks. In this regard, we introduce two extensions to our general algorithm.

### 3.2.1 Shock from outside the banking system

A shock from outside the banking system, say from sector $Mg$, is reflected by a deterioration of the respective credit risk parameters, namely the probability of default $PD_{Mg}$ and the loss given default $LGD_{Mg}$, which are assigned to the exposure $Mg_i$ bank $i$ has to the concerned sector. We denote such a deterioration by $\Delta PD_{Mg}$ and $\Delta LGD_{Mg}$. As a result, bank's Tier 1 capital ratio decreases and its RWA increases, because the bank's portfolio is exposed to higher expected and unexpected credit losses (depending on the bank's individual size of the exposure $Mg_i$).

The new levels of bank's total assets, Tier 1 capital ratio and RWA after the shock from outside the banking system at $t = 0$ is given by

$$\begin{aligned}
TA^s_{i,0} &= TA_{i,0} - \Delta LGD_{Mg} \cdot \Delta PD_{Mg} \cdot Mg_i \\
Tier1^s_{i,0} &= Tier1_{i,0} - \Delta LGD_{Mg} \cdot \Delta PD_{Mg} \cdot Mg_i \\
\Delta RW^s_{i,0} &= max\{0, RW(\Delta PD_{Mg}, \Delta LGD_{Mg}, 1)\} \\
RWA^s_{i,0} &= RWA_{i,0} + \Delta RW^s_{i,0} \cdot Mg_i.
\end{aligned}$$

In the formulae we use the superscript $s$ to indicate that the variable results from a shock outside the banking system. We use $\Delta RW^s_{i,0}$ to refer to the change in the risk weight of the exposure $Mg_i$ due to the changed $PD_{Mg}$ and $LGD_{Mg}$. The corresponding percentage change in Tier 1 capital ratio is given by

$$\Delta_0 ln(CapRat_i^s) = ln(CapRat_{i,0}^s) - ln(CapRat_{i,0}) = ln(\frac{Tier1_{i,0}^s}{RWA_{i,0}^s}) - ln(\frac{Tier1_{i,0}}{RWA_{i,0}}).$$

The resulting change in Tier 1 capital ratio translates into an increase in bank's own PD according to $\beta_{caprat}$ from the logit-regression.

At $t = 1$ the extended version of the algorithm determines the bank-specific shocked PDs. We obtain

$$P_1(i|A) = \begin{cases} 1 & \text{if } CapRat_{i,t} = 0 \\ min\{1, P_0(i|A) + \beta_{caprat}(P_0(i|A) - P_0(i|A)^2)\Delta_0 ln(CapRat_i^s)\} & \text{else} \end{cases}$$

Banks' PDs given by these formula describe the direct impact of the losses that arise from an adverse development in sector $Mg$. The PDs generated in the subsequent iteration steps (ie $t \geq 2$) are then calculated according to the general algorithm above. In this way, the algorithm shows how the shock from sector $Mg$ propagates through the interbank network and results in higher PDs for banks. Finally, the banking system loss induced by the shock from sector $Mg$ can be derived according to the expression (3). Importantly, $BSLoss$ does only take into account the contagion effect between banks, which is initiated from the shock in sector $Mg$. In other words, any possible direct adverse effects of the shock reflecting a reduction in total assets net of loan loss allowances between $t = 0$ and $t = 1$ are excluded from $BSLoss$.

### 3.2.2 Capital surcharges to curb contagion risks

Capital requirements can be designed in such a way that they provide an incentive for banks to internalize negative externalities arising from interconnectedness in their ex-ante decision-making. We propose an extension to our algorithm, which may help to assess the effectiveness of capital surcharges to curb contagion risks. In principle, the source of the initial shock could be from inside or outside the banking system. In the following, we focus on a shock from outside the banking system, namely from sector $Mg$.[2] As a buffers size we propose $k$ percentage points times the share of exposure to sector $Mg$ in total bank assets to mitigate contagion risks stemming from this sector. That means, if a bank's portfolio would solely consist of assets to sector $Mg$ the increase in the capital ratio would be $k$ percentage points. The new capital is given by

$$Tier1_{i,0}^P = Tier1_{i,0} + 0.0k \cdot RWA_{i,0} \cdot \frac{Mg_i}{TA_{i,0}}.$$

In the formula we use the superscript $P$ to indicate that the variable is set by the regulator. Instead of the original capital ratio we may use the capital ratios after imposing the bank-specific buffer as a starting point for the algorithm, ie $CapRat_{i,0}^P = \frac{Tier1_{i,0}^P}{RWA_{i,0}}$. As a result, the new Tier 1 capital ratio

[2] In principle, $k$ could also be applied to exposure banks have to other banks to take into account the costs of contagion stemming from the banking system.

translates into a decrease in bank's own PD according to $\beta_{caprat}$ from the logit-regression which we may use as the starting PD in the algorithm. To evaluate the effectiveness of the introduction of an additional capital buffer we can then rely on the expression (3). In this regard, we compare the banking system loss according to alternatives values for $k > 0$ with the banking system loss according to the base case for $k = 0$.

# 4 Policy applications

As outlined in chapter 3 our algorithm can be used in various kinds of analyzes related to macroprudential supervisory policies. In the area of macroprudential supervision it is essential to have detailed knowledge on the relative importance of single banks in the system. Supervisory resources might be assigned according to this relative importance. Also, the measurement of consequences for the banking system of a shock stemming from disturbances in a sector from outside the banking system, is an objective for supervisory activities.

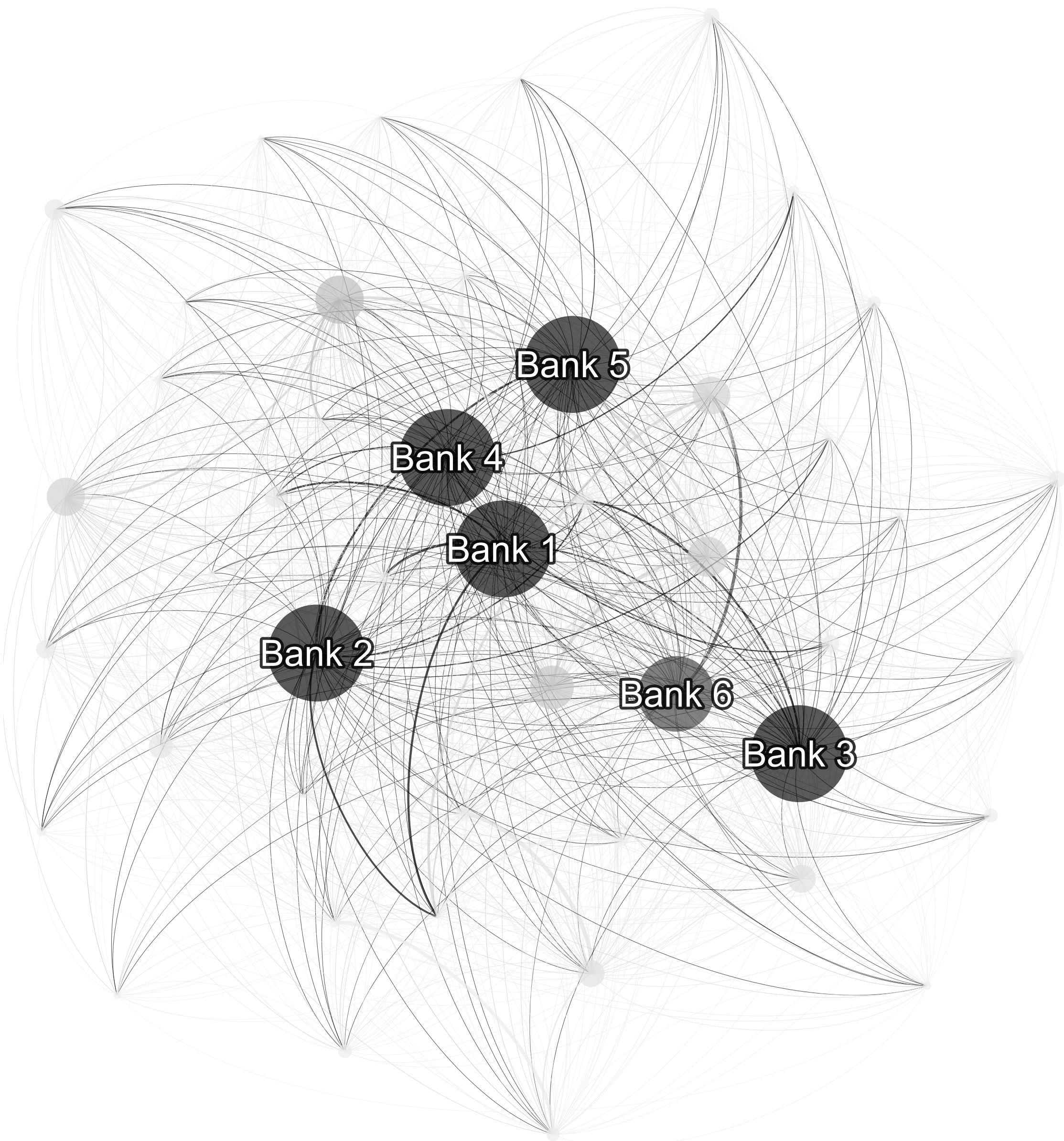
For macroprudential supervisory policy makers it is equally important to be in the position to assess the consequences of certain policy measures they plan to implement. Our measure also has the advantage to obtain economically interpretable cardinal differences between the benefits of different scales of policy intervention. In this section we will present two exemplary policy applications of the algorithm including an assessment of the benefits of certain policy measures that might be implemented.

## 4.1 Data

The performed analysis is based on year-end data for 2013. Interbank credits are taken from Deutsche Bundesbank's credit register of large exposures and loans of 1.5 million € or more. The further variables used in the algorithm stem from various sources: Tier1-Capital, Total Assets and RWAs are taken from German banks' reporting systems to the Deutsche Bundesbank, such as the Common Reporting Framework (CoRep) and the Banking Statistics (BiSta). The unconditional PDs in $t = 0$ for German banks are derived from their credit ratings assigned by the three major rating agencies, namely Fitch Ratings, Moody's, and Standard & Poor's. In order to combine the information obtained from the three rating agencies, the average of the historically observed default rates per credit ranking is calculated. If a bank is not rated by one of the three major rating agencies then a standard rating in investment grade range is assumed. For the estimation of the elasticity parameter $\beta$, we follow the approach presented by Craig, Kötter, and Krüger (2014) and obtain $\hat{\beta}_{caprat} = -2.005332$. Details and results of the logistic regression can be found in Appendix A2.

The network resulting from these inputs can be described as follows. It is a directed graph (or '*digraph*') with $1,710$ nodes (German banks) and $20,425$ arcs (single credits from one bank to another). The average in-degree, ie the average number of credits a representative bank of the system receives, is $11.94$, ranging from $0$ to $1,357$ credits, and the average out-degree, the average number of credits

Figure 1: Illustration of the network




Note: For illustrative purposes the graph for the bank network does not include all institutions used in the analysis. Instead, we focus on banks whose default generates banking system losses amounting to at least 1 billion € in the analysis of section 4.2, ie the graph contains $46$ nodes and $1,188$ edges. The label, the size and the color of the nodes depends on the banking system losses stemming from the application in Table 1. The thickness of the edges represents the size of the respective credit and the edges' color is determined by the node color of the bank having this credits as liability in its books.

a representative bank gives, is $11.94$, ranging from $0$ to $997$ credits. In order to give a short insight into the density of our network, two kinds of measures are consulted. Firstly, the average path length in the network[3] is $2.283$ with a diameter, ie the largest distance in terms of paths between any two nodes in the network, of $5$. Secondly, the average clustering coefficient[4] is $0.714$. The latter is a measure of cliquishness in a network ranging from $0$ (very loose network) to $1$ (very dense network). Figure 1 shows an extract of the network we use.

### 4.2 Systemic importance of single institutions

The first possible application of the algorithm we present is the simulation of single institutions' defaults. This analysis provides the possibility to rank institutions with respect to the banking system loss their default induces in the banking system. The resulting ranking is of a cardinal nature, ie the differences in the banking system loss between single institutions are economically interpretable. To take account of the confidentiality of the data bank's identity is masked and the $BSLoss$ calculated for each bank is normalized in the application.

For the calculation of the ranking we simulate the default of each of the $1,710$ institutions in the network, ie we change the $PD_i$ of the respective institution at the beginning of the algorithm to $PD_1 = 1$. Thus, the algorithm is repeated $1,710$ times to derive the banking system loss the default of every single institution causes. The propagation of the exogenous shock stops when the changes in the PDs of all counterparties are smaller than a threshold value $\epsilon = 0.000001$.

For this application we apply the following default criteria: A bank is considered as defaulted if its Tier 1 capital ratio falls below $6\%$[5] or its leverage ratio falls below $1.5\%$. Table 1 displays an excerpt of the obtained rankings for the top 20 banks in the network whose default causes the greatest banking system loss.[6] One interesting fact that can be concluded from Table 1 is that the first five banks can be considered as almost equally important for the German banking system with regard to interconnectedness. A default of one of these banks would result in a $BSLoss$ of up to $15.9\%$ of the sum of *BSLoss* of all banks in the network. In fact, a default of one of these five banks would result in a mutual default of the remaining four banks due to their high level of interconnectedness. Furthermore, it can be seen that for many of the displayed banks a significant proportion of the total banking system loss stems from indirect propagation in the subsequent rounds following the direct

[3]In the network literature a path is defined as a sequence of links connecting two more or less distant nodes such that no node is hit twice. This fact distinguishes a path from a walk, since in the course of a walk every node in the network can be hit several times.

[4]The intuition behind the clustering coefficient can be illustrated by a simple example: imagine three nodes of a network *a,b* and *c* with arcs *ab* and *ac*. The clustering coefficient gives the probability that there also exists an arc *bc*.

[5]The default criterion follows Memmel and Sachs (2011). It reflects the minimum capital requirements of the Basel III framework to be from 2015 onwards.

[6]Our results are driven among other factors by the loss given default (LGD) of banks. For our calculations we chose the value widely used in the respective literature as LGD$= 0.45$. It should be noticed that this value in combination with the sensitive default criteria chosen might overestimate the resulting banking system loss since a lower share of a failing institutions' bankrupt estate might be ultimately lost in practice.

Table 1: Ranking resulting from the algorithm

| | Total effect | | | Direct effect - Round 2 - | | Indirect effect - Following rounds - | | |
|---|---|---|---|---|---|---|---|---|
| Rank | $\frac{\text{BSLoss}_i^T}{\text{BSLoss}_1^T}$ | # rounds | $\frac{\text{defaults}_i^T}{\text{defaults}_1^T}$ | $\frac{\text{BSLoss}_i^{dir}}{\text{BSLoss}_i^T}$ | % defaults$_i^T$ | $\frac{\text{BSLoss}_i^{ind}}{\text{BSLoss}_i^T}$ | % defaults$_i^T$ | $\frac{\text{BSLoss}_i^T}{\text{Credits}_i^{rec}}$ |
| 1 | 100.0% | 16 | 100.0% | 5.2% | 2.8% | 94.8% | 97.2% | 8.64 |
| 2 | 100.0% | 14 | 100.0% | 5.4% | 4.1% | 94.6% | 95.9% | 8.34 |
| 3 | 100.0% | 14 | 100.0% | 8.1% | 3.9% | 91.9% | 96.1% | 5.52 |
| 4 | 100.0% | 14 | 100.0% | 6.2% | 3.8% | 93.8% | 96.2% | 7.20 |
| 5 | 100.0% | 10 | 100.0% | 9.6% | 4.6% | 90.4% | 95.4% | 4.68 |
| 6 | 34.5% | 11 | 69.5% | 36.4% | 79.1% | 63.6% | 20.9% | 1.23 |
| 7 | 10.8% | 10 | 2.0% | 47.2% | 46.4% | 52.8% | 53.6% | 0.95 |
| 8 | 8.6% | 11 | 1.5% | 59.0% | 71.4% | 41.0% | 28.6% | 0.76 |
| 9 | 7.1% | 9 | 11.9% | 65.0% | 97.6% | 35.0% | 2.4% | 0.69 |
| 10 | 6.3% | 10 | 0.8% | 46.0% | 50.0% | 54.0% | 50.0% | 0.98 |
| 11 | 6.1% | 9 | 0.4% | 95.6% | 33.3% | 4.4% | 66.7% | 0.47 |
| 12 | 3.3% | 9 | 0.5% | 92.3% | 71.4% | 7.7% | 28.6% | 0.49 |
| 13 | 3.0% | 10 | 1.4% | 71.7% | 50.0% | 28.3% | 50.0% | 0.62 |
| 14 | 2.4% | 8 | 0.2% | 99.0% | 66.7% | 1.0% | 33.3% | 0.45 |
| 15 | 1.9% | 8 | 0.3% | 64.5% | 75.0% | 35.5% | 25.0% | 0.70 |
| 16 | 1.8% | 8 | 0.4% | 81.6% | 66.7% | 18.4% | 33.3% | 0.55 |
| 17 | 1.5% | 9 | 1.5% | 75.9% | 77.3% | 24.1% | 22.7% | 0.59 |
| 18 | 1.4% | 6 | 0.2% | 98.4% | 66.7% | 1.6% | 33.3% | 0.46 |
| 19 | 1.3% | 8 | 0.3% | 64.1% | 75.0% | 35.9% | 25.0% | 0.70 |
| 20 | 1.2% | 8 | 0.5% | 85.3% | 71.4% | 14.7% | 28.6% | 0.53 |

*Note: Credits$_i^{rec}$ is the total amount of interbank liabilities of bank $i$ with respect to the German banking system. Columns 1 and 3 display the BSLoss and the number of defaults relative to the values for Bank 1. The third column gives the number of rounds the algorithm realizes for every single default scenario. Columns 5 and 7 display the proportion of the total BSLoss in the respective rounds following the shock and columns 6 and 8 give the percentage of defaulting banks in the respective round as fraction of the total number of defaulting banks.*

propagation effect in round 2. Therefore, we argue that the persistence of contagion can form a crucial part in the analysis of the consequences certain institutions' defaults may have on the entire banking system.

We compare the ranking derived from $BSLoss$ to rankings obtained from calculations for other established measures for interconnectedness. To this end, Table 2 displays the Spearman's rank correlation coefficient $\rho$ between $BSLoss$ and the method to determine domestic systemically important banks ($D - SIBs$[7]), the $\alpha$-centrality measure from Bonacich and Lloyd (2001) and the in-degree.

The second column of Table 2 refers to the total score of the indicator-based method to determine D-SIBs. This method comprises four different dimensions: size, interconnectedness, complexity and

[7]Based on proposal by Basel committee to determine domestic systemically important banks.

Table 2: Rank correlation coefficients between $BSLoss$ and other measures for interconnectedness

| | D-SIBs (Total score) | D-SIBs (Intercon.) | Bonacich centrality | In-Degree measure |
|---|---|---|---|---|
| $\rho$ | 0.39 | 0.66 | 0.96 | 0.70 |

substitutability. The third column refers to the sub-score for interconnectedness, which is mainly based on the volume of interbank credits. In the fourth column the rank correlation with the Bonacich centrality measure is displayed. This eigenvector-based measure takes into account the entire network structure[8]. The results show that the rankings are (partly closely) correlated between $BSLoss$ and the other measures for interconnectedness.[9] This is consistent to the fact that $BSLoss$ has features of recursive centrality measures, such as the Bonacich eigenvector-based centrality measure, and also takes into account the amount of interbank credits, such as the method to determine the level of interconnectedness for D-SIBs. However, in contrast to these two measures for interconnectedness $BSLoss$ takes additionally into account the credit quality of interbank exposures and the inherent relationship between the PD of the debtor bank and the PD of the creditor bank.

In a further step we simulate a policy intervention in the form of the implementation of a SIFI-buffer to the banking system. We introduce a surcharge on the initial capital-ratio of the first six banks in our ranking. To each of the six institutions we assign a rise in the initial capital-ratio by $k$ percentage points in the following way:

$$\frac{Tier1^P_{i,0}}{RWA_{i,0}} = \frac{Tier1_{i,0}}{RWA_{i,0}} + 0.0k$$

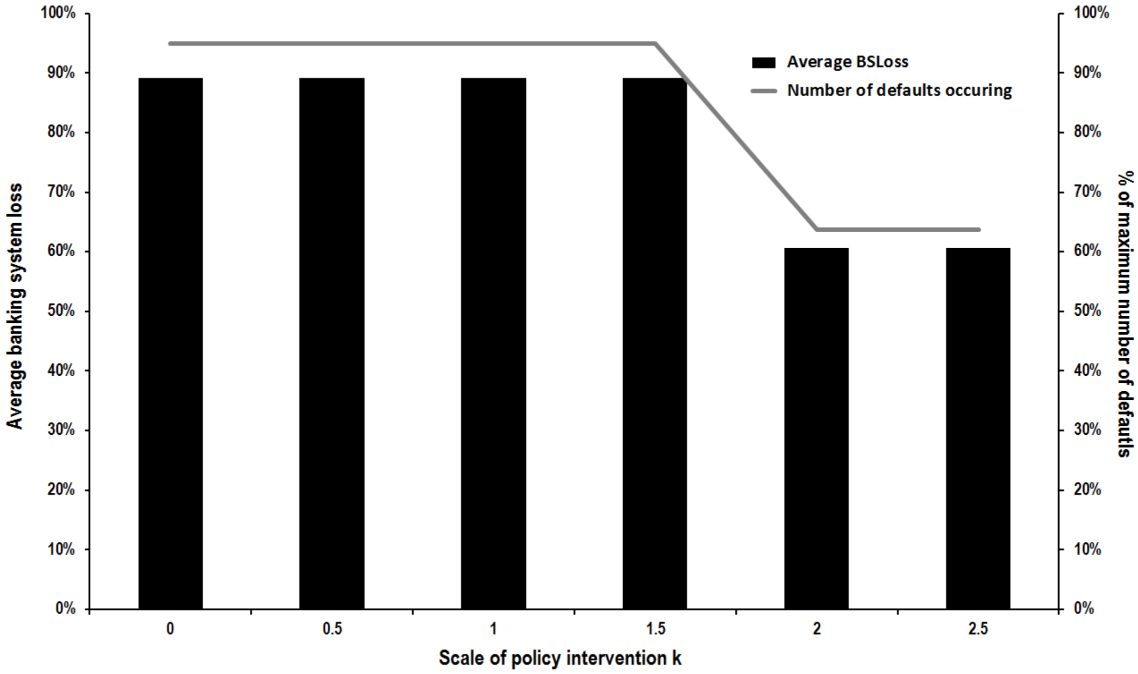
To compare the outcomes of this policy intervention with the initial scenario, we display the average $BSLoss$ due to a default of these six institutions in the system with values for the policy parameter $k$ ranging from 0 to 2.5 in Figure 2. Furthermore, on the secondary axis, we display the average number of defaulting institutions due to a default of these six banks. Since in practice a SIFI-buffer will not exceed 2.5 percentage points, we think that these values for $k$ are adequate.

Figure 2 illustrates the usefulness of a SIFI-buffer in this framework. The implemented SIFI-buffer needs to exceed a certain value (here: $k \geq 2$) in order to prove useful in curbing losses in the respective system. In case the SIFI-buffer is lower than that value, certain banks will default despite its stronger capital base. This default causes nearly the same disturbances as it does without a SIFI-buffer.

[8]In addition we have weighted the respective adjacency matrix with the corresponding interbank matrix.

[9]The ranking between $BSLoss$ and the Bonacich centrality measure is highly correlated ($\rho = 0.96$). The rank correlation is weaker for other reporting dates (eg $\rho = 0.83$ based on year-end 2012 data).

Figure 2: Average banking system loss for different scales of policy intervention




*Note: The vertical axes represent the average BSLoss (primary axis) and the average number of defaults (secondary axis) the defaults of the six most 'important' institutions cause in the system. These values are displayed relative to the BSLoss and the defaults for 'Bank 1' in Table 1. The policy parameter k on the horizontal axis is the percentage point change in the capital-ratio the first 6 institutions from our ranking in Table 1 need to apply.*

### 4.3 Collapse of a housing bubble

The second application for the algorithm we present is a scenario analysis on the consequences of a decline of prices in the real estate market. In a first step we calculate the banking system loss caused by such a decline in real estate prices. In a second step we assess the usefulness of a policy intervention of different strength, namely an increase of the capital-ratio for all banks ordered by the regulator. For different scales of this regulatory requirement we can observe how the banking system losses develop and assess the usefulness of the respective intervention.

For the calculations in this application we follow the formulae presented in section 3.2 in combination with the standard algorithm from section 3.1. The data used is essentially the same as in section 4.2 with an additional variable for the absolute amount of credits every bank holds in the real estate sector. This variable is taken from the German banks' reporting system to the Deutsche Bundesbank. Furthermore, the probability of default for mortgage loans is set to $PD_{Mg} = 0.015$[10] and is not affected by the shock.

As in the application before, an exogenous shock hits the system, this time causing an increase

[10]This value represents an average which has been determined on the basis of supervisory reports for a representative selection of German banks.

of the PDs of some institutions in the system. The shock consists of an increase in the loss given default (Δ LGD) of credits to the real estate sector. This disturbance influences the values of Tier1-capital and the RWAs and, consequently, changes the capital-ratio of the respective bank. Every institution engaged in the real estate sector will therefore experience a rise in its PD by a certain amount depending on the change in the capital-ratio. Hence, the initial increase of the PD for a certain bank depends on the institutions' extent of engagement in the real estate sector. After the implementation of this shock into the initial round $t = 1$ of the algorithm according to section 3.2, the propagation of the shock in the network takes place exactly in the same manner as before using the algorithm from section 3.1 from round $t = 2$ on.

The results in this application are dependent on the parameter $k$. This parameter determines the scale of policy intervention. Before any shock hits the system, all institutions are forced to increase their capital-ratio by $k$ percentage points relative to its engagement in the real estate sector. Hence, the required change in the capital-ratio is

$$\frac{Tier1^P_{i,0}}{RWA_{i,0}} = \frac{Tier1_{i,0}}{RWA_{i,0}} + 0.0k \cdot \frac{Mg_i}{TA_{i,0}} \tag{4}$$

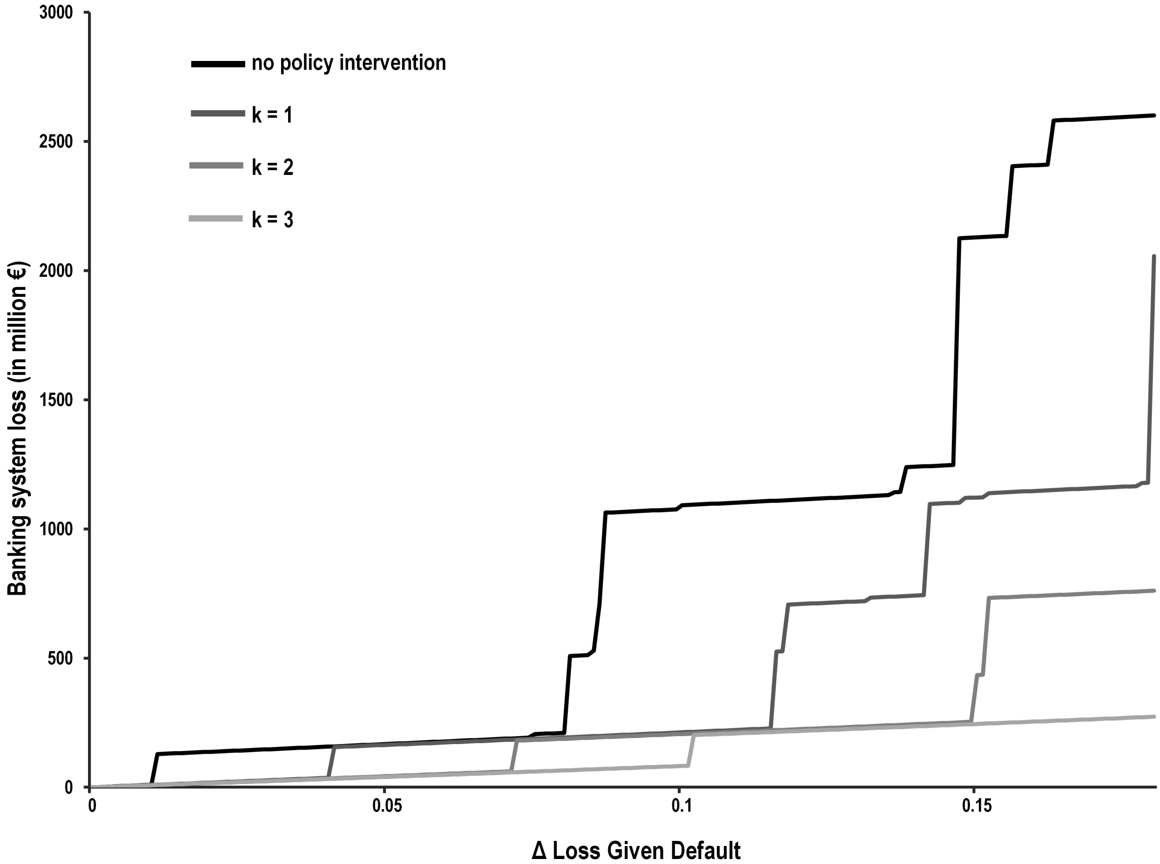
with $Tier1^P_{i,0}$ as the required level of Tier1-Capital the banks need to reach for equation (4) to hold.

Figure 3 displays the results for the algorithm applied to different scenarios. We perform the algorithm for four scenarios with $k$ ranging from 0 (ie no policy intervention) to 3. For every single of these scenarios we calculate the banking system loss (on the vertical axis) for different values of declines in house prices in form of a change in loss given default ranging from $\Delta$LGD$=\ 0$ to $\Delta$LGD$=\ 0.18$ (on the horizontal axis). This banking system loss is calculated as the difference between total assets in round $T$ and round 1 according to equation (3). These values contain the contagion effects occuring in the banking system only, excluding the direct adjustments taking place as a response to the initial price shock.

Figure 3 illustrates the reduction in the $BSLoss$ a certain policy intervention entails compared to the baseline case of no policy intervention for a given ΔLGD. Consider for example the case of $\Delta$LGD$= 0.15$, ie a rise in the loss given default of 15%: a policy intervention of $k = 1$ will mitigate the consequences of the exogenous shock in the system compared to the situation without regulatory intervention. Additionally, it can be deduced from figure 3 that a stronger intervention, eg $k = 2$, would further reduce the scale of consequences for the system. The banking system loss prevailing from the given reduction in house prices might be reduced by $\sim\ 1$ bn. € with an intervention of $k = 1$ and by an additional $\sim 0.5$ bn. € with an intervention of $k = 2$.

In order to assess the *BSLoss* stemming from interconnectedness of banks only, disregarding the initial effect on the institutions engaged in the real estate sector, the algorithm provides the possibility to separate these two effects. Figure 5 illustrates these differences for the case of no policy intervention (ie $k = 0$) and the scales of ΔLGD introduced before. It can be seen that the initial effects on banks (the difference between the black and the grey line) have about the same size as the banking system

Figure 3: Illustration of different scales of policy intervention for several shock scenarios



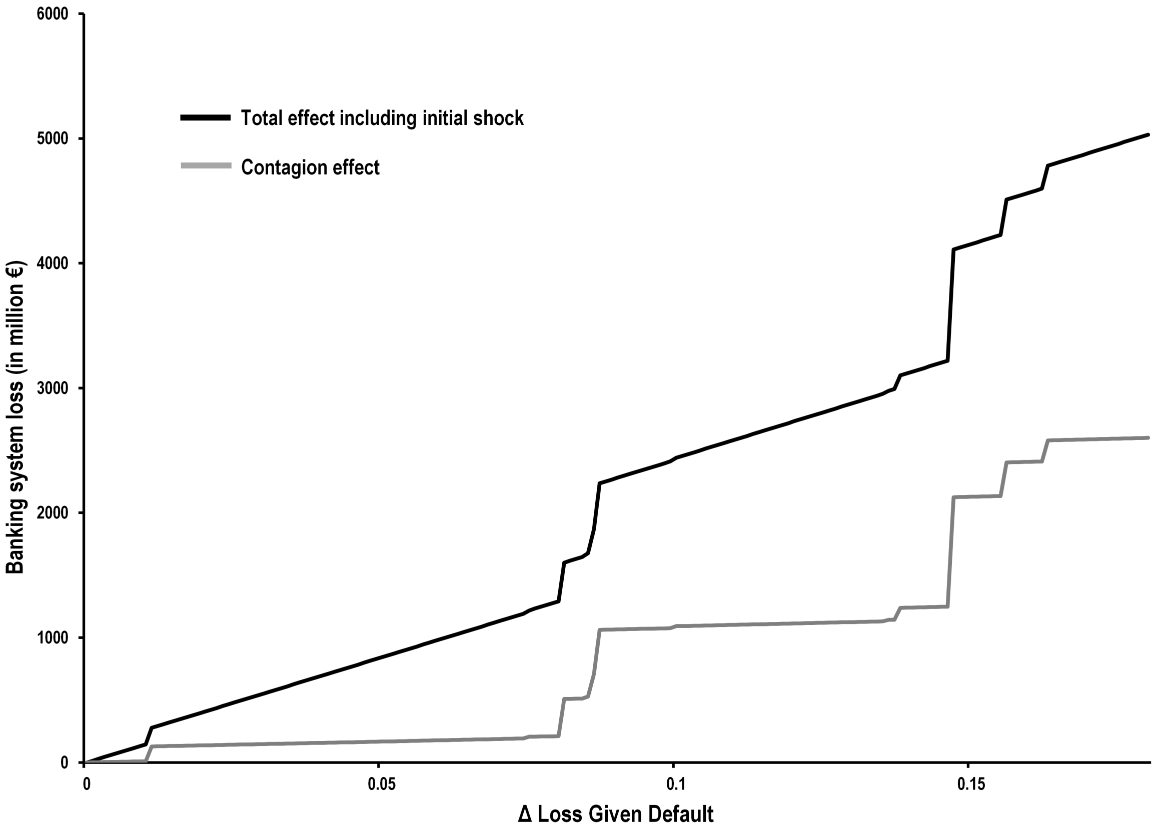

losses stemming from interconnectedness in the network (the absolute amount of the grey line).[11] This indicates that, apart from the direct effects, banks are equally hit by the indirect contagion effects in the financial system following the decline in house prices. Furthermore, Figure 5 shows the lines are not increasing steadily, but follow a pattern with alternating sharp and moderate increases. This represents the strong relative impact of defaults for the banking system.

# 5 Conclusion

In this paper we have developed an analytical framework to quantify the cost of contagion in the banking system. The proposed measure $BSLoss$ describes the increase n expected credit losses due to contagion risk. It combines several merits from other established measures of interconnectedness (eg risk sensitive, expressible in monetary units). This framework may prove useful in the context

[11]The total effect of the house price decline (black line) is calculated as $BSLoss_{total} = \sum_j (TA_{j0} - TA_{jT})$ and the contagion effect (grey line) as $BSLoss_{contagion} = \sum_j (TA_{j1} - TA_{jT})$.

Figure 4: Illustration of the scale of the propagation effect for different scenarios




of macroprudential surveillance and policy evaluation. In particular, it may be used to study the effectiveness of different macroprudential instruments.

We apply the proposed analytical framework to two very important fields of macroprudential supervision: firstly, we come up with a ranking according to the relative importance of banks in their banking system. Secondly, we quantify the benefits of two macroprudential policy instruments: capital surcharges for systemically important financial institutions (a SIFI-Buffer) and a capital surcharges for mortgages to curb contagion.

However the limitations of the model must be considered when interpreting the results. As with all partial analysis, we have to keep in mind that our analysis is a ceteris paribus analysis. While a shock might propagate faster than interbank credits can change, leaving our analysis valid, financial institutions might rearrange their portfolios and change their lending and borrowing habits in response to a policy change, exposing our policy evaluation to the Lucas critique.

The framework can also be extended to assess the interconnectedness in other financial networks, such as the shadow banking system. A more sophisticated risk analysis might identify relevant shock scenarios and feed those into the proposed algorithm. Other types of contagion channels (eg liquidity channel) and their interaction among each other would be also of interest for the analysis. Most likely, one would also like to take into account the cost of capital surcharges in terms of reduced lending, as was done by Kashyap and Stein (2004).

Finally, the model may react very sensitively to certain parameter conditions. For example, small changes in LGD may result in big jumps in $BSLoss$ and the number of defaulted banks ("cliff-effect"). However, it should be noted the sensitive dependency on initial conditions can hardly be avoided because it reflects an inherent problem when making predictions of the outcome of complex, non-linear dynamic network systems.

# 6 Appendix

**A1: Calculation of Risk-weights, Tier 1 Capital and Total Assets**

Risk-weights are calculated using the IRB formula

$$RW(PD, LGD, M) = 1.06 \cdot 12.5 \cdot LGD \cdot \left( \mathcal{N}\left( \frac{\mathcal{N}^{-1}(PD) + \sqrt{\rho(PD)}\mathcal{N}^{-1}(q_{99.9\%})}{\sqrt{1-\rho(PD)}} \right) - PD \right), \cdot \frac{1 + b(PD) \cdot (M - 2.5)}{1 - 1.5 \cdot b(PD)}$$

where $b(PD) = 0.11852 - 0.05478 \cdot ln(PD)$ and $\rho$ is the *asset correlation*, which is defined by

$$\rho(PD) = \frac{1 - e^{-50PD}}{1 - e^{-50}} \cdot 0.12 + (1 - \frac{1 - e^{-50PD}}{1 - e^{-50}}) \cdot 0.24.$$

Total assets and Tier 1 capital are calculated net of loan loss allowance. For practical reasons, we assume the loan loss allowance can be described approximately by the expected loss based on regulatory risk parameters, ie the loan loss allowance charged to bank $i$ on the interbank credit $W$ granted to bank $j$ in round $t$ equals to:

$$LLA_{i,t} = W_{ij} \cdot P_t(i|A) \cdot LGD$$

As a result, the total assets in $t+1$ which has changed due to a change in probability of default of the debtor banks $j$ amounts to:

$$\begin{aligned} TA_{i,t+1} &= TA_{i,t} - \sum_j (LLA_{i,t} - LLA_{i,t-1}) \\ &= TA_{i,t} - \sum_j W_{ij} \cdot LGD \cdot (P_{t-1}(j|A) - P_{t-2}(j|A)) \end{aligned}$$

In principle, the same reasoning applies to Tier 1 capital, which we approximately describe by the difference between total assets and debt liabilities $D$. Therefore, Tier 1 capital of bank $i$ in round $t$ equals to:

$$Tier1_t = TA_t - D$$

Using the procedure for $TA$ above and assuming $D$ is measured at the (constant) notional value of the debt liabilities, Tier 1 capital in $t+1$ amounts to:

$$Tier1_{i,t+1} = Tier1_{i,t} - \sum_j W_{ij} \cdot LGD \cdot (P_{t-1}(j|A) - P_{t-2}(j|A))$$

### A2: Logistic regression

In order to estimate by how many percent points the probability of default changes if the capital ratio of a bank changes by one percent, we run the following logistic regression:

$$Pr(default) = p(ln(CapRat), X) = F(\alpha + \beta ln(CapRat) + \gamma X) \tag{5}$$

where $F(z) = e^z/(1 + e^z)$ is the cumulative logistic distribution and $\pi$ is the probability of the dependent variable equalling a "success", in our case this is a default. Taking the natural logarithm on both sides and rearranging terms, we get the inverse of the logistic function, the logit:

$$\ln \frac{p(ln(CapRat), X)}{1 - p(ln(CapRat), X)} = \alpha + \beta ln(CapRat) + \gamma X. \tag{6}$$

However, we are not interested in effect on the odds ratio, $\frac{p(ln(CapRat),X)}{1-p(ln(CapRat),X)}$, but rather the probability of default, $p(ln(CapRat), X)$. Therefore, we use the natural exponential function on both sides of the equation and rearrange:

$$p = \frac{e^{\alpha+\beta ln(CapRat)+\gamma X}}{1 + e^{\alpha+\beta ln(CapRat)+\gamma X}}. \tag{7}$$

In order to know by how many percent points the probability of default changes if the capital ratio changes by one percent, we have to take the derivative with respect to the capital ratio:

$$\begin{aligned}\frac{\delta p}{\delta ln(CapRat)} &= \frac{(1 + e^{\alpha+\beta ln(CapRat)+\gamma X})\beta e^{\alpha+\beta ln(CapRat)+\gamma X} - \beta e^{2(\alpha+\beta ln(CapRat)+\gamma X)}}{(1 + e^{\alpha+\beta ln(CapRat)+\gamma X})^2} \\ &= \beta \frac{e^{\alpha+\beta ln(CapRat)+\gamma X}}{(1 + e^{\alpha+\beta ln(CapRat)+\gamma X})^2}.\end{aligned}$$

Now, from equation (7) we deduce that $e^{\alpha+\beta ln(CapRat)+\gamma X} = \frac{p}{1-p}$ and hence,

$$\frac{\delta p}{\delta ln(CapRat)} = \beta \frac{\frac{p}{1-p}}{(1 + \frac{p}{1-p})^2} = \beta(p - p^2) \tag{8}$$

which describes by how many percent points the probability of default changes if the capital ratio changes by one percent.

The regression results for the capital-ratio as independent variable are included in Table 3.

Table 3: Regression results

| Variables | Capital-Ratio | |
|---|---|---|
| Constant | -29.73419 | |
| | (0.965) | |
| log(Tier1-Capital over RWA) | **-2.005332** | *** |
| | (0.000) | |
| Depreciation and Adjustments over Equity | -0.0014031 | |
| | (0.729) | |
| Administration Expenses over Total Assets | 0.0165785 | ** |
| | (0.050) | |
| Return on Equity | -0.0765523 | *** |
| | (0.000) | |
| Cash and overnight Interbank Loans over Total Assets | 0.0314922 | *** |
| | (0.002) | |
| Log Total Assets | 0.3088996 | *** |
| | (0.000) | |
| (pseudo) $R^2$ | 0.1355 | |

Note: The regression is based on a panel-dataset containing 8288 observations and 6 periods (from 2001 to 2006). We control for regional fixed effects.

**A3: Update of the PDs in the contagion algorithm**

For infinitesimally small changes of $ln(CapRat)$ we can write

$$\mathrm{d}p = \beta(p - p^2) \cdot \mathrm{d}ln(CapRat)$$

or, equivalently,

$$\frac{\mathrm{d}p}{p - p^2} = \beta \cdot \mathrm{d}ln(CapRat).$$

Taking into account that $\int \frac{\mathrm{d}p}{p(p-1)} = ln\left(\frac{p}{p-1}\right) + c$, with a constant $c$, and applying simple rules for the logarithm, we obtain

$$ln\left(\frac{p_t}{p_t - 1}\right) = \beta(ln(CapRat_t) - ln(CapRat_{t-1})) + ln\left(\frac{p_{t-1}}{p_{t-1} - 1}\right),$$

which is equivalent to equation (2) in the main text. Note that we assume that the PD is a function of $CapRat$ only and that all other explanatory variables in the regression do not have any influence on the PD.

### A4: Academic example

We assume a complete interbank credit market where each bank is connected to all other banks. Three uniform banks exist, each with $PD_i = 0.02$, $TA_i = 20$, $RWA_i = 10$, $CAP_i = 1$ and $W_{i,j} = 2$ for $i$,$j \in \{1,2,3\}$. Table 4 displays the quantity $BSLoss$ for different levels of shocks and over various rounds. From the simulation results it can be seen that $BSLoss$ is monotonically increasing in $\varphi$. $BSLoss$ has a discontinuity on the interval between $\varphi = 820bp$ and $\varphi = 830bp$. Above a certain threshold the default of banks leads to jumps in the PDs and $BSLoss$. It can be seen that any additional increment exceeding that shock level does not lead to a further increase of $BSLoss$. Any further increase of the initial shock would only result in an earlier default of these banks, leaving the overall $BSLoss$ unchanged. The corresponding sequences of the PDs are shown in Figure 5.

Table 4: Development of $BSLoss$ for different shocks over the rounds of the algorithm

| Round | $\varphi = 400bp$ | $\varphi = 600bp$ | $\varphi = 820bp$ | $\varphi = 830bp$ | $\varphi = 1000bp$ | $\varphi = 1200bp$ |
|---|---|---|---|---|---|---|
| 1 | 0.09674 | 0.14063 | 0.18766 | 0.18977 | 0.22544 | 0.26685 |
| 2 | 0.12194 | 0.17985 | 0.24312 | 0.24598 | 0.29455 | 0.35133 |
| 3 | 0.13539 | 0.20034 | 0.27159 | 0.27482 | 0.32963 | 0.39379 |
| 4 | 0.14517 | 0.21638 | 0.29531 | 0.29891 | 0.36011 | 3.85929 |
| 5 | 0.15114 | 0.22636 | 0.31034 | 0.31419 | 3.81087 | 5.29200 |
| 6 | 0.15513 | 0.23339 | 0.32146 | 0.32551 | 5.29200 | |
| 7 | 0.15766 | 0.23801 | 0.32902 | 0.33322 | | |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | | |
| 13 | 0.16199 | 0.24679 | 0.34500 | 0.34959 | | |
| 14 | 0.16211 | 0.24708 | 0.34566 | 3.77945 | | |
| 15 | 0.16219 | 0.24729 | 0.34613 | 5.29200 | | |
| 16 | 0.16224 | 0.24742 | 0.34646 | | | |
| ⋮ | ⋮ | ⋮ | ⋮ | | | |
| 22 | 0.16232 | 0.24768 | 0.34715 | | | |
| 23 | | 0.24769 | 0.34718 | | | |
| 24 | | 0.24769 | 0.34720 | | | |
| 25 | | 0.24770 | 0.34721 | | | |
| ⋮ | | | ⋮ | | | |
| 29 | | | 0.34724 | | | |

Figure 5: Development of the banks' PDs for different shock sizes

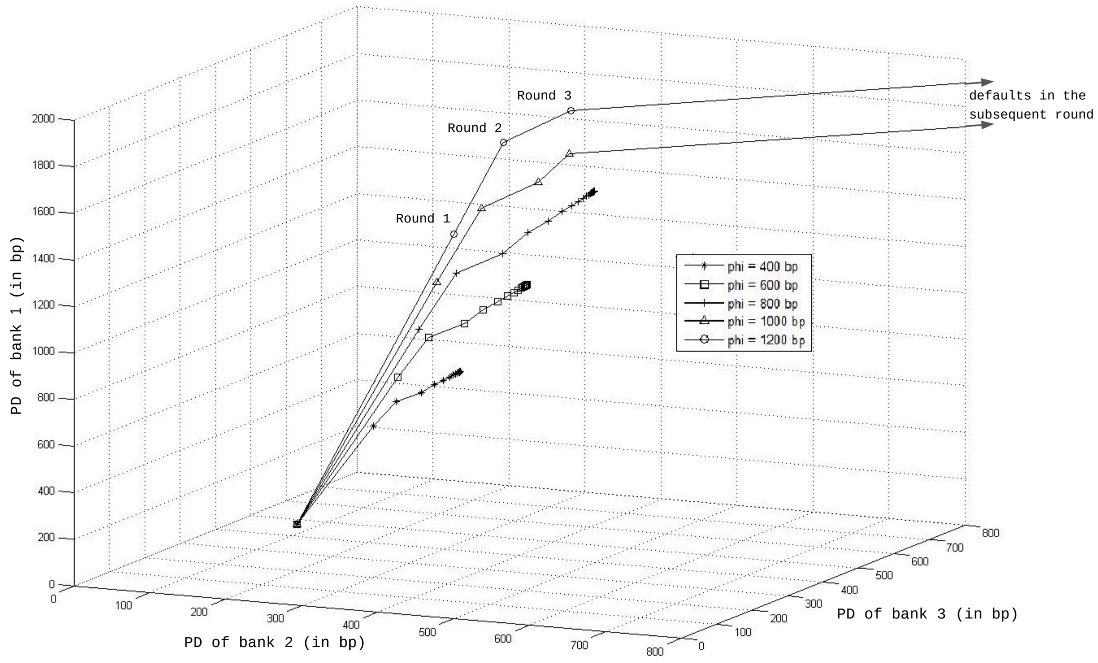



Figure 6: Development of the banks' PDs for different values of the initial PD-vectors

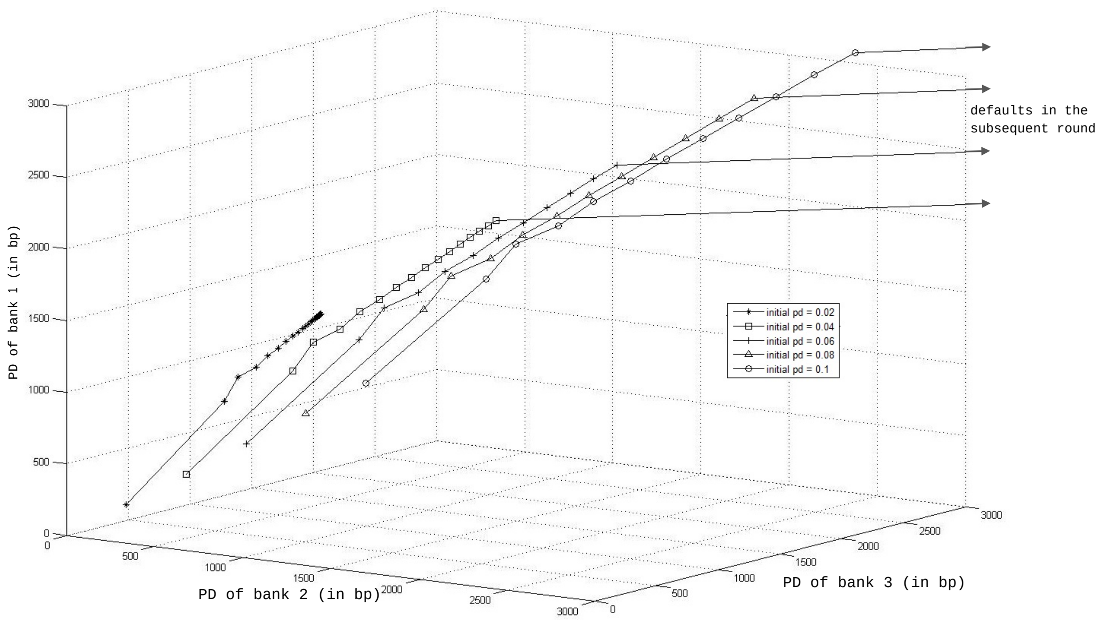



Table 5 displays the quantity $BSLoss$ for different initial PD-vectors and over various rounds given a shock ($\varphi = 500bp$) has hit one bank. From the simulation results it can be seen that $BSLoss$ is not a monotonically increasing function on the closed interval from $PD_0 = 0.02$ to $PD_0 = 0.06$. For low levels, an increase in initial PDs results in higher $BSLoss$ due to the fact of the logarithmic relationship between $CapRat$ and $PD$. At $PD_0 = 0.031$ banks' default results in a jump of the PDs and $BSLoss$. Notably, any additional increment exceeding that $PD_0$ level does not lead to a further increase in $BSLoss$, but actually a decrease. Given that the PDs are bounded by 1, any additional increase in initial PDs (ie $PD_0 > 0.031$) results in a corresponding reduction in $BSLoss$ ceteris paribus. The corresponding development of stressed PDs depending on the level of initial PDs is illustrated in Figure 6.

Table 5: Development of $BSLoss$ for different values of the initial PD-vectors

| Rounds | $PD_0 = \begin{pmatrix}0.02\\0.02\\0.02\end{pmatrix}$ | $PD_0 = \begin{pmatrix}0.03\\0.03\\0.03\end{pmatrix}$ | $PD_0 = \begin{pmatrix}0.031\\0.031\\0.031\end{pmatrix}$ | $PD_0 = \begin{pmatrix}0.04\\0.04\\0.04\end{pmatrix}$ | $PD_0 = \begin{pmatrix}0.06\\0.06\\0.06\end{pmatrix}$ |
|---|---|---|---|---|---|
| 1 | 0.13537 | 0.14140 | 0.14195 | 0.14639 | 0.15395 |
| 2 | 0.18270 | 0.19524 | 0.19639 | 0.20594 | 0.22249 |
| 3 | 0.21586 | 0.23611 | 0.23800 | 0.25390 | 0.28216 |
| 4 | 0.24401 | 0.27239 | 0.27509 | 0.29801 | 0.33968 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 11 | 0.33166 | 0.41209 | 0.42059 | 0.49952 | 0.67325 |
| 12 | 0.33658 | 0.42290 | 0.43217 | 0.51937 | 3.62869 |
| 13 | 0.34059 | 0.43239 | 0.44239 | 0.53786 | 4.96800 |
| 14 | 0.34385 | 0.44073 | 0.45144 | 0.55519 | |
| 15 | 0.34651 | 0.44807 | 0.45947 | 0.57151 | |
| 16 | 0.34869 | 0.45456 | 0.46661 | 3.66112 | |
| 17 | 0.35046 | 0.46031 | 0.47298 | 5.07600 | |
| 18 | 0.35191 | 0.46540 | 0.47867 | | |
| ⋮ | ⋮ | ⋮ | ⋮ | | |
| 35 | 0.35822 | 0.50139 | 0.52123 | | |
| 36 | 0.35826 | 0.50207 | 3.67364 | | |
| 37 | 0.35829 | 0.50269 | 5.12460 | | |
| 38 | 0.35832 | 0.50324 | | | |
| ⋮ | ⋮ | ⋮ | | | |
| 47 | 0.35842 | 0.50627 | | | |
| 48 | | 0.50646 | | | |
| ⋮ | | ⋮ | | | |
| 83 | | 0.50814 | | | |

## References


F. Allen and D. Gale. Financial contagion. *The Journal of Political Economy*, 108, 2000.

S. Battiston, M. Puliga, R. Kaushik, P. Tasca, and G. Caldarelli. Debtrank: Too central to fail? financial networks, the fed and systemic risk. *scientific reports*, 2:541, 2012.

BCBS. Global systemically important banks: updated assessment methodology and the higher loss absorbency requirement. *Committee publications*, 2013.

BIS, IMF, and OECD. G-10 report on financial sector consolidation. 2001.

P. Bonacich. Factoring and weighting approaches to status scores and clique identification. *Journal of Mathematical Sociology*, 2, 1972.

M. Brunnermeier, L. Clerc, and M. Scheicher. Otc derivatives: new rules, new actors, new risks. *Financial Stability Review Banque de France*, 17:123–134, 2013.

B. Craig, M. Kötter, and U. Krüger. Interbank lending and distress: Observables, unobservables, and network structure. *Discussion Paper Deutsche Bundesbank*, 2014.

ECB. Financial stability review. 2012.

L. Eisenberg and T. H. Noe. Systemic risk in financial systems. *Management Science*, 47, 2001.

X. Freixas, B. Parigri, and J.-C. Rochet. Systemic risk, interbank relations and liquidity provision by the central bank. *Journal of Money, Credit and Banking*, 32, 2000.

A. A. Jobst. Measuring systemic risk-adjusted liquidity (srl) - a model approach. *IMF Working Paper*, 12/209, 2012.

A. K. Kashyap and J. S. Stein. Cyclical implications of the basel ii capital standards. *Economic Perspectives*, 10, 2004.

A. Landherr, B. Friedl, and J. Heidemann. A critical review of centrality measures in social networks. *Business and information systems engineering*, 2, 2010.

S. Martinez-Jaramillo, B. Alexandrova-Kabadjova, B. Bravo-Benitez, and J. P. Solorzano-Margain. An empirical study of the mexican banking system's network and its implications for systemic risk. *Journal of Economics Dynamics and Control*, 1, 2014.

C. Memmel and A. Sachs. Contagion in the interbank market and its determinants. *Discussion Paper Deutsche Bundesbank*, 17/2011, 2011.

P. E. Mistrulli. Assessing financial contagion in the interbank market: Maximum entropy versus observed interbank lending patterns. *Working Paper Banca d' Italia*, 2011.

F. Packer and N. Tarashev. Rating methodologies for banks. *BIS Quarterly Review*, June, 2011.

L. C. G. Rogers and L. A. M. Veraart. Failure and rescue in an interbank network. *Management Science*, 59, 2012.

C. Upper. Simulation methods to assess the danger of contagion in interbank markets. *Journal of Financial Stability*, 7, 2010.