Are SME Loans Less Risky than Regulatory Capital Requirements Suggest?*

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Abstract

Our paper addresses firm size as a driver of systematic credit risk in loans to small and medium-sized enterprises (SMEs). Key contributions are the use of a unique data set of SME lending by over 400 German banks and relating systematic risk to the size dependence of regulatory capital requirements. What sets our sample apart is its comprehensive coverage of the particularly rich and well-developed credit market for SMEs in Germany. We estimate asset correlations as the key measure of systematic risk from historical default rates. Our results suggest that systematic risk tends to increase with firm size, conditional on the respective rating category. We also compare the size of this effect with the capital relief that has been granted in Basel II for SMEs relative to large firms. Our asset correlation estimates suggest a significantly larger relative difference from the corresponding values for large firms than reflected in the regulatory capital requirements in two cases: first, for SME loans in the corporate portfolio of the Internal Ratings-Based Approach and, second, for SME loans treated under the revised standardized approach of Basel II.

Keywords: Asset Correlation, Basel II, Minimum Capital Requirements, Single Risk Factor Model.

JEL classification: G 21, G 33, C 13.

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Our paper belongs to a well-established strand of empirical work on the systematic risk in loans to small and medium-sized enterprises (SMEs). We explore in particular the dependence of systematic risk on firm size and compare the size of this effect with the capital relief granted to SME lending in the regulatory minimum capital requirements of Basel II.[1] Another key contribution is the use of a unique data sample of SME lending by over 400 German banks. What sets this sample apart is its comprehensive coverage of the particularly rich and well-developed credit market for SMEs in Germany, the availability of banks' internal ratings, and the capture of the recent financial crisis in the time series. Asset correlation is used as the key measure of systematic risk. It also drives the systematic risk in the Asymptotic Single Risk Factor (ASRF) model of Gordy [2003] that is the basis for the regulatory minimum capital requirements in the Internal Ratings-Based Approach (IRBA) of Basel II.

The asset correlation is estimated in the first step from historical default rates by the Maximum-Likelihood (ML) estimator of Gordy and Heitfield [2002]. In the second step and based on the asset correlation estimates, the dependence of capital requirements on firm size is compared both with the dependence implicit in the current IRBA risk weight functions and with risk weights in the Revised Standardized Approach (RSA).[2] We address the question if the regulatory capital for SMEs reflects the systematic risk correctly relative to other asset classes. Instead of the level of regulatory capital, we consider the relative reduction in systematic risk which is measured as a capital requirement in the ASRF model, with respect to large firms. In this way, our study also contributes to the empirical question of an appropriate (relative) calibration of regulatory capital requirements for SME lending. Since regulatory capital requirements can affect the interest margins required by the lender, only their appropriate calculation in the sense that they reflect the actual risk posed by the borrower will ensure an optimal credit supply for the economy. In many countries, such as Germany, SMEs are the backbone of the economy. Therefore, appropriate capital requirements are crucial for economic growth. If the regulatory capital requirements were increased, bank competition would decrease and higher concentration would lead to a decrease in the number of bank-SME relationships. In the end, SME lending would be reduced (see Mercieca et al. [2009]).

For our analysis it is important to separate a potentially higher *firm-specific* (idiosyncratic) risk of SMEs – that is typically reflected in higher default probabilities – from a potentially lower *systematic* risk of SMEs. Since capital requirements in the ASRF model refer by construction only to systematic risk, lower asset correlations (and therefore lower systematic risk) compared with large firms would *ceteris paribus* also suggest lower capital requirements for SMEs. The capital requirements for an SME loan in the IRBA depend on both the default probability and the risk weight function, which in turn depends on the asset correlation value. As a consequence, lower systematic risk for SMEs can well be in line with higher capital requirements for SMEs if

SMEs have higher default probabilities, i.e. higher firm-specific risk, than large firms.
Numerous research studies exist that use either historical default rates or equity prices to estimate asset correlations for different regulatory asset classes, as summarized and analysed in Berg et al. [2011]. These studies generally estimate lower values than the ones used in the IRBA.
Our empirical results confirm previous findings that asset correlations increase with firm size conditional on the rating category. Furthermore, they suggest that the relative differences between the capital requirements for large corporates and those for SMEs (in other words, the capital relief for SMEs) are, in two cases, smaller in the current regulatory framework than suggested by our empirically estimated asset correlations: (1) In the IRBA the empirically observed potential for increasing the difference in capital requirements between SME loans in the corporate portfolio and large corporates might amount up to 24 percentage points depending on firm size. This could be achieved by adjusting the asset correlation parameters of the IRBA formula. For SMEs in the IRBA retail portfolio, however, there is no empirical indication supporting a change in the current minimum capital requirements. (2) For all loans assigned to the SME portfolio in the RSA, the empirical results suggest a significantly higher relative reduction compared to large firms than reflected in the current capital requirements. The capital relief potential amounts to values between 15 and 35 percentage points. Before the capital relief reflected in these figures is translated into a policy message to adjust the current regulatory capital requirements, several caveats also need to be considered; these caveats are described in the last section.

Relative Calibration

In the previous section we stated that an evaluation of regulatory capital requirements should distinguish between the *level* of capital and the *relative* difference against other asset classes. In the development of Basel II the second aspect, often referred to as *relative calibration*, was addressed first. It is a key aspect of regulatory capital requirements because it ensures that banks *ceteris paribus* have to hold more (less) capital for a more (less) risky asset or, in other words, that the right incentives are given for a bank's risk management. The *level calibration* was instead guided by the requirement to keep the overall level of capital in the international banking system broadly constant when transitioning from Basel I to Basel II. This was achieved in an iterative top-down calibration, guided by several *quantitative impact studies* coordinated by the Basel Committee on Banking Supervision.
In this study, we consider only the *relative* calibration since the appropriate *level* of regulatory capital cannot be satisfactorily assessed for the following two reasons: (1) The overall level of capital requirements was determined in the top-down calibration of the whole Basel II framework, also involving for example the 99.9% confidence level of the value-at-risk, the

scaling factor of 1.06 for credit-risk-weighted assets, and the benchmark maturity of 2.5 years. There is no reason to believe that this very different calibration goal will provide asset correlations similar to the estimates from time series of default rates. (2) Gordy and Heitfield [2000] and Düllmann et al. [2010] show that asset correlation estimations can generate significant downward biases when the underlying time series of default rates are short. Through a relative comparison of asset correlation estimates for large companies with SMEs, both of which are affected by this estimation bias, we expect to mitigate the impact of this effect.

Our analysis is very much in the spirit of previous analyses that were carried out for the relative calibration of Basel II. The asset correlations are estimated based on the ASRF model underlying the IRBA capital requirements. We use large corporates as a *benchmark*, which means that they are assumed to be correctly calibrated in level. This is motivated by the fact that the Basel Committee on Banking Supervision has spent substantial effort on calibrating these portfolios due to its immense economic importance. Then we compare the relative difference of both capital requirements based on estimated asset correlations and the current IRBA capital requirements from the capital requirements for this benchmark. Comparing these two relative differences can provide useful information for an evaluation of the capital relief for SMEs granted in Basel II.

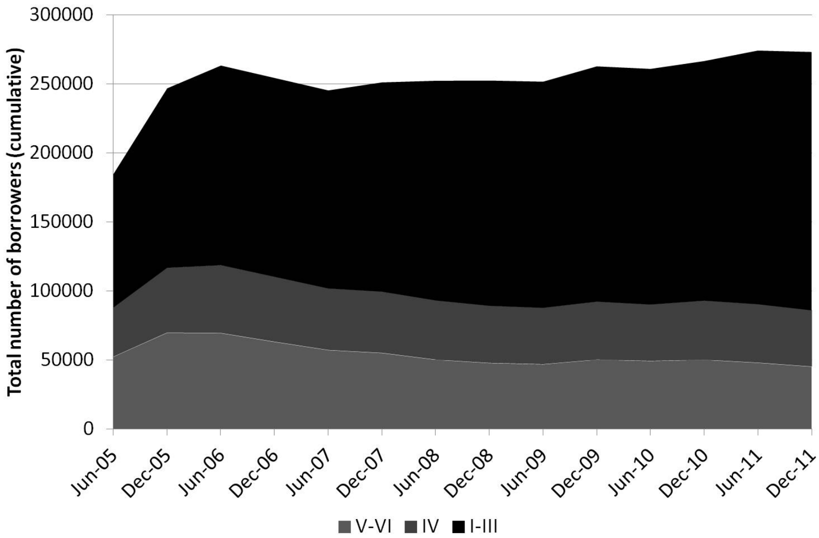
Data

The data that have been provided by more than 400 small and large German banks cover the time period from 1 January 2005 to 31 December 2011. Therefore, the sample includes as large a part of the entire SMEs and large corporates as possible. The considered time horizon has been chosen in order to apply the Basel II definition of default throughout the entire observation period (see Basel Committee on Banking Supervision [2006]). In each considered time period our sample on average includes approximately 250,000 rated borrowers. Although the vast majority of banks have adopted the RSA, their rating systems have been designed in line with the requirements of an IRBA rating system. The considered creditors include all domestic firms (except credit institutions) for which an IRBA PD was available. Retail and specialized lending are not considered.

The sample is broken down into size and rating buckets, as asset correlations in the IRBA risk weight functions are dependent both on the PD and the firm size. Default rates for certain rating-size buckets exhibit semi-annual seasonality due to banks' provisioning policy. Thus, the underlying time series have to be seasonally adjusted. Since the data set contains only 14 half-yearly observations for every rating-size bucket, it is difficult to identify the seasonal pattern. In this regard, we apply the difference-from-moving-average method to seasonally adjust the time series. In order to avoid overadjustment during the financial crisis (second half of 2008, both halves of 2009), these outliers are excluded from the estimation of the moving average.

In our sample every observation includes two figures: the number of borrowers at the beginning of the respective period in the respective bucket, and the number of defaults up to the end of that period. Buckets are built in three dimensions: yearly turnover, rating, and time.[3] If a borrower is included in the credit portfolio at the beginning of the semi-annual horizon and its credit is redeemed in the following half-year, the credit is counted as 0.5 for the number of credits. Nevertheless this effect is minor. The chart in Exhibit 1 shows the number of borrowers with respect to the rating category. The increase in numbers in the first two semi-annual periods is clearly due to the step-by-step adoption of the rating methodology by the banks in 2004–2005. The fluctuation of the number of defaults is indicative for "hybrid" PDs, i.e. PDs that are based on a mix between a pure point-in-time and a pure through-the-cycle approach. However, the "hybrid" PDs in our data set are closer to point-in-time PDs.

Exhibit 1: **Semi-annual number of borrowers and defaults with respect to rating category**



The rating buckets are defined according to the size and the rating of the borrower. Rating categories are determined by a master scale drawn up by the Joint Banking Initiative for the Financial Location of Germany [2010] (IFD). As every bank determines its own rating categories with respect to number and labeling, the IFD combines the different rating categories into one master scale. The purpose was to improve the transparency of IRBA ratings and to enable borrowers to compare their own rating across banks. This master scale comprises six rating categories I to VI with I being the best rating. The different rating systems of each bank in Germany can be converted into the IFD scale. The sample period of seven years covers the recent global financial crisis and roughly comes close to include a full credit cycle. However, due to the robustness of foreign demand from outside Europe and due to special national arrangements to support the German economy (for example, extensive use of flexible time arrangements in order to avoid lay-offs and the temporary introduction of the car scrappage premium), the sample period does not capture a period of severe recession in the SME sector (see Exhibit 2(b)).

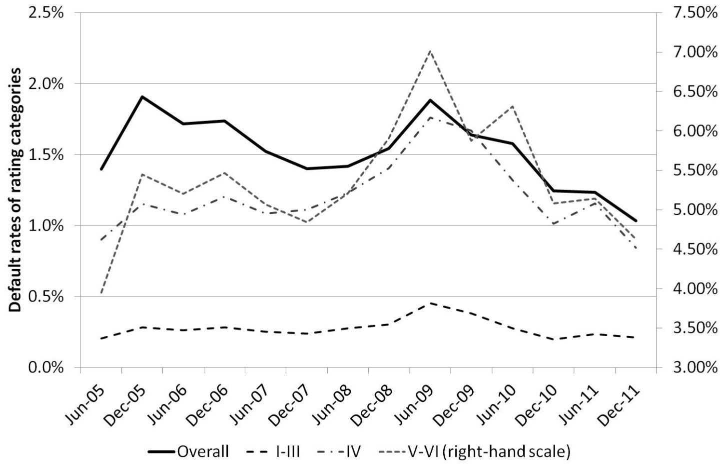
The sample in total contains 14 semi-annual periods in the time dimension. As the data are pooled across banks it is important to ensure that double counting of defaults due to two or more banks granting loans to the same customer is avoided. In our sample, the vast majority of banks adhere to the "regional principle" which states that banks are only allowed to serve customers within a specified region. Therefore, there should be no significant distortion from the double counting of defaults and the estimations should not be significantly biased. This has been confirmed by various robustness checks.

The size of the borrowers is proxied by the turnover which is published in the balance sheet of each borrower. The following six size categories measured in € million are chosen: [0; 0.3], (0.3; 1], (1; 2.5], (2.5; 5], (5; 50], and (50; $+\infty$).

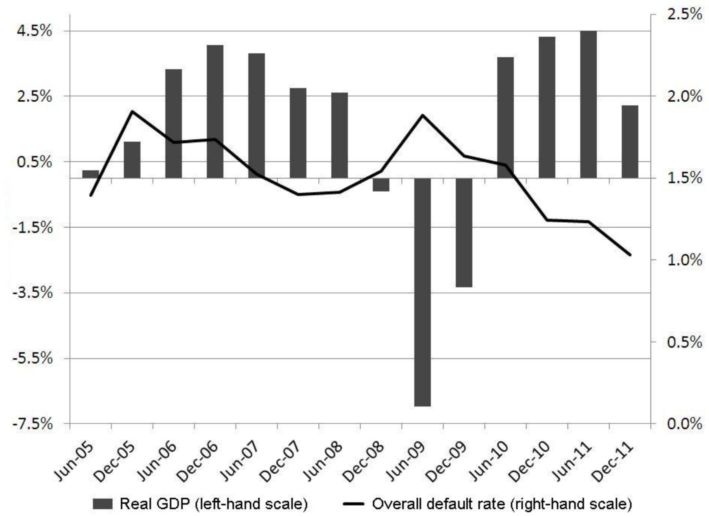
Exhibit 2 illustrates the evolution of each risk category (upper chart) and the evolution of the default rates over all rating classes compared to the real GDP (lower chart) over the time horizon under consideration. The highest default rates for German SMEs are observed in 2005, well before the financial crisis started. During the peak of the crisis in 2009, however, the default rates only rose moderately compared to the severe drop in real GDP. This observation supports the view that although the business cycle clearly affects default rates of SMEs, German SMEs weathered the financial crisis quite well.

Exhibit 2: **Semi-annual default rates in percent with respect to rating category and real GDP**

(a) Default rates by rating category



(b) Default rates and real GDP (annual growth rates)



Methodology

Model

The analysis is based on the widely known ASRF model of Gordy [2003] that is also the foundation of the IRBA risk weight functions for credit exposures in the banking book. Default is triggered in this model if the ability-to-pay process Y_i of firm i falls below an exogenous default threshold γ_i. Y_i follows a standard normal distribution. It can be decomposed into the return of a systematic and unobservable factor X and an idiosyncratic firm-specific part ε_i:

$$Y_i = \sqrt{\rho_i} \cdot X + \sqrt{1 - \rho_i} \cdot \varepsilon_i.$$

X and ε_i are independent for every obligor i and follow a Gaussian distribution. The factor loading $\sqrt{\rho_i}$ of the systematic risk factor can be interpreted either as the sensitivity against systematic risk or as the square root of the asset correlation ρ_i. For this analysis the common assumption of a constant ρ_i is applied, which is typical for such empirical studies as it allows this parameter to be estimated from a cross section and the index i is dropped. The Bernoulli variable L_i describes whether a credit event has occurred during the considered horizon $(L_i = 1)$ or not $(L_i = 0)$. It is important to differentiate between the unconditional and the conditional default probability. The unconditional default probability of obligor i for the time period t is defined as follows:

$$P(L_i = 1) = P(Y_i < \gamma_i) = \Phi(\gamma_i)$$

where Φ denotes the cumulative distribution function of a standard normal distribution. Since homogeneity in the obligor buckets is assumed, the index i for the distance to default γ_i of a specific firm is dropped.

In this study the ML estimator advocated in Gordy and Heitfield [2002] is applied when retrieving the main results. The ML estimator is flexible enough to allow for the possibility that obligors in different rating and size buckets may be sensitive to different risk factors. For robustness tests we employ a Method-of-Moments (MM) estimator[4] and also use annual time periods in addition to semi-annual ones for computing the default rates (see Appendix B–D). The estimation methodology is described in Appendix A.

Capital Requirements

Since we are ultimately concerned with the calibration of capital requirements we do not consider only the asset correlation estimates but also capital requirements dependent on these estimates. More precisely, we consider the "empirical risk weight function", i.e. the risk weight function based on the empirically estimated asset correlations $\hat{\rho}$, rather than the asset correlation estimates themselves, in order to assess the calibration of the IRBA capital requirements:

$$RW^{Est}(\hat{\rho}, PD) = 1.06 \cdot 12.5 \cdot LGD \cdot \left[\Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\hat{\rho}}x^*_{99.9\%}}{\sqrt{1-\hat{\rho}}}\right) - PD\right] \cdot f(M, PD)$$

where LGD denotes the Loss Given Default, $x^*_{99.9\%}$ the 99.9% quantile of the standard normal distribution function and $f(M, PD)$ the maturity adjustment dependent on the effective maturity M and the PD with $f(M, PD) = (1 + (M - 2.5) \cdot b(PD))/(1 - 1.5 \cdot b(PD))$ and $b(PD) =$

$(0.11852 - 0.05478 \cdot \log(PD))^2$. The LGD is set to 0.45 and the maturity M to 2.5 years in our analysis.

The current Basel II capital requirements are calculated according to the IRBA formulae for corporate exposures:

$$\begin{aligned} RW^{BII}(PD,S) &= 1.06 \cdot 12.5 \cdot LGD \cdot \left[\Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho(PD,S)} x^*_{99.9\%}}{\sqrt{1-\rho(PD,S)}} \right) - PD \right] \\ &\quad \cdot f(M,PD). \end{aligned}$$

Turnovers above € 50 million are lumped together in a single bucket since the risk weight curve would remain flat above this turnover threshold (for a constant PD). For a turnover above € 2.5 million we have applied the corporate risk weight function including the capital relief due to the turnover dependence of the asset correlation:

$$\rho(\text{PD,S}) = 0.24 - (0.24 - 0.12) \cdot \left(1 - e^{-50\text{PD}}\right) - 0.04 \left(1 - \frac{\min\{50, \max\{S, 5\}\} - 5}{45}\right)$$

with the last term of the function being the size adjustment for SMEs.
The retail risk weight curve (Other Retail) has been applied for a turnover below € 2.5 million. Analyses of the Bank for the Accounts of Companies Harmonised (BACH) database from the European Committee of Central Balance Sheet Data Offices support the consideration of the first three turnover classes as Other Retail since the average ratio of turnover to liabilities of credit institutions amounts to 3.1 in 2009 and € 1 million is the exposure threshold for the retail portfolio. The Retail risk weight curve differs from the one for corporate exposures as it does not dependent on the effective maturity M and size S. The corresponding asset correlation is lower than the one for the corporate portfolio and ranges from 3% to 16%:

$$\rho(PD) = 0.16 - (0.16 - 0.03) \cdot \left(1 - e^{-35\text{PD}}\right).$$

In both cases, the capital charge is determined by multiplying the exposure at default with the risk weight and the solvability coefficient of 0.08.
The risk weights in the RSA are not based on models. More precisely, they are determined by a simple step function with 100% for loans in the corporate portfolio without an external rating and 75% for loans in the retail portfolio. This construction implies that the RSA risk weights are only partially risk-sensitive. In Germany SMEs typically do not have external ratings.

Results

Asset Correlation Estimates

In order to evaluate the relative calibration we need to consider the fact that besides firm size measured by yearly turnover, credit quality, i.e. the rating, has also been found to be a potential driver of the estimation of asset correlations (e.g. Hahnenstein [2004] and Düllmann and Scheule [2006]). This two-dimensional dependency is also reflected in the current IRBA risk weight functions. Therefore, we estimate the asset correlation for a matrix of rating and turnover buckets. This procedure enables us to compare capital requirements for different size buckets depending on the rating with the respective IRBA capital requirements. The estimation results are presented in Exhibit 3. Since the time periods in the sample cover six months we transform the estimates of a half-year PD_h using the formula $PD = 1 - (1 - PD_h)^2$ into PDs for a one-year horizon. This transformation is necessary for the analysis of the capital requirements since PDs in Basel II always refer to a one-year horizon.

Exhibit 3: **ML estimates for asset correlations and PDs (in percent)**

Asset correlation estimates						
Rating category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	0.51	0.59	0.62	0.66	0.81	1.71
	(0.26)	(0.26)	(0.27)	(0.32)	(0.34)	(0.80)
IV	0.50	0.43	0.62	0.74	0.70	1.72
	(0.26)	(0.20)	(0.28)	(0.37)	(0.32)	(0.93)
V–VI	0.56	0.31	0.49	0.64	0.80	1.54
	(0.22)	(0.13)	(0.20)	(0.28)	(0.32)	(0.81)
PD estimates (one-year horizon)						
Rating category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	0.66	0.56	0.56	0.56	0.49	0.42
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)
IV	2.11	2.33	2.54	2.70	2.48	2.56
	(0.12)	(0.12)	(0.15)	(0.18)	(0.16)	(0.28)
V–VI	10.08	10.52	11.31	10.69	9.72	8.97
	(0.41)	(0.32)	(0.43)	(0.48)	(0.49)	(0.71)

Standard errors determined analytically from asymptotic Fisher information matrices are given below in brackets.

The asset correlation estimates in Exhibit 3 tend to increase with firm size when holding the rating constant. This increase, however, is not perfectly monotonic and is more pronounced in some rating categories than in others. The level of asset correlations never exceeds two percent and is on average considerably below the asset correlations in the IRBA capital requirements. A possible underestimation of the asset correlations could result from the fact that for each size and rating bucket the correlations were estimated for well-diversified portfolios with respect to business sectors. Since the time series used is still relatively short at seven years, it is open to question whether one full business cycle is captured in the estimations. If this is not the case, negative biases may arise in the estimation of the asset correlations (see for instance Gordy and Heitfield [2000], Dietsch and Petey [2004], and Düllmann and Scheule [2006]).

Evaluation of IRBA Capital Requirements

By comparing the size dependence of "estimated capital requirements" (i.e. based on empirical asset correlation estimates) with the size dependence "hard-wired" into the corresponding IRBA capital requirements we seek to answer the question of whether the size dependence of IRBA capital requirements is appropriate in light of the new empirical results. For this purpose, and for different size buckets, we consider the relative difference of the (estimated and Basel II) capital requirements from the corresponding capital requirements of "large" corporates (i.e. firms with a yearly turnover higher than €50 million) which serve as a benchmark.[5] If both (relative) differences are negative (indicating a capital relief) and if the absolute value of the difference for the empirical estimates is higher than that of the difference for the regulatory numbers, this may be interpreted as an indication that our empirical results *ceteris paribus* would support lower Basel II capital requirements for SMEs. Exhibit 4 shows the estimated capital requirements and the Basel II ones in terms of risk weights.

Exhibit 4: **Capital requirements in terms of risk weights per rating class (in percent)**

Estimates						
	Other Retail			Corporate		
Turnover (€ m) / Rating category	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	4.0	3.9	4.0	4.2	4.3	6.4
IV	9.6	9.4	12.6	14.6	13.2	23.9
V–VI	30.3	22.6	30.2	33.9	36.3	50.8
Basel II						
	Other Retail			Corporate		
Turnover (€ m) / Rating category	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	39.8	36.6	36.6	61.2	62.4	67.8
IV	62.3	63.6	64.8	100.9	107.7	130.3
V–VI	80.3	81.4	83.6	159.7	167.1	196.5

In order to determine the differences in capital requirements we calculate the relative difference between each risk weight in all turnover classes up to €50 million and the corresponding risk weight for the largest turnover class which serves as a benchmark. As an example, consider the relative difference between the estimated risk weight of turnover class (5, 50] and turnover class

>50 in rating category I-III:

$$\Delta_{5-50}^{Est,I-III} = \frac{RW_{5-50}^{Est,I-III} - RW_{>50}^{Est,I-III}}{RW_{>50}^{Est,I-III}} = \frac{4.3\% - 6.4\%}{6.4\%} = -32.8\%.$$

The same is done for the Basel II risk weights:

$$\Delta_{5-50}^{BII,I-III} = \frac{RW_{5-50}^{BII,I-III} - RW_{>50}^{BII,I-III}}{RW_{>50}^{BII,I-III}} = \frac{62.4\% - 67.8\%}{67.8\%} = -8.0\%.$$

Doing this for each risk weight gives us the relative differences for both the estimated and the Basel II capital requirements in Panel A and B of Exhibit 5:

Exhibit 5: **(Relative) differences of IRB capital requirements based on asset correlation estimates and of Basel II capital requirements from the benchmark, ordered by rating and turnover class (in percent)**

Panel A: For capital requirements based on asset correlation estimates						
	Other Retail			Corporate		
Rating category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	-37.3	-39.1	-37.3	-34.6	-32.8	0.0
IV	-59.9	-60.6	-47.5	-38.9	-45.0	0.0
V–VI	-40.4	-55.5	-40.5	-33.3	-28.5	0.0
Panel B: For Basel II IRB capital requirements						
	Other Retail			Corporate		
Rating category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	-41.3	-46.0	-46.0	-9.8	-8.0	0.0
IV	-52.2	-51.2	-50.3	-22.6	-17.4	0.0
V–VI	-59.1	-58.6	-57.5	-18.7	-15.0	0.0
Panel C: Total differences between A and B						
	Other Retail			Corporate		
Rating category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	3.9	6.9	8.7	-24.8	-24.8	0.0
IV	-7.7	-9.4	2.8	-16.4	-27.5	0.0
V–VI	18.8	3.0	17.0	-14.5	-13.5	0.0

Next we compare the differences of capital requirements from the benchmark between both cases, i.e. the ones based on asset correlation estimates and the ones based on the Basel II formulae. For this purpose, we compute the total differences between the relative differences of the estimated capital requirements and the relative differences based on the Basel II formulae. Again using the example of turnover class (5, 50] and rating category I-III, the total difference of capital requirements amounts to

$$\Delta_{5-50}^{I-III} = \Delta_{5-50}^{Est,I-III} - \Delta_{5-50}^{BII,I-III} = -32.8\% - (-8.0\%) = -24.8\%.$$

The results for all total differences are given in Panel C of Exhibit 5. The total differences vary to some extent, which means that it is difficult to draw general conclusions from this representation. Thus, for an overall assessment of these results, we average the total differences of the capital requirements by weighting them with the number of loans per rating class. For example, the average total difference in capital requirements for the size category $(5, 50]$ is obtained as

$$\Delta_{5-50}^{T} = \Delta_{5-50}^{Est} - \Delta_{5-50}^{BII} = -33.9\% - (-10.0\%) = -23.9\%.$$

Exhibit 6 contains the weights for every turnover class and Exhibit 7 the resulting average total differences:

Exhibit 6: **Mean weights for ratings per turnover class (in percent)**

	Other Retail			Corporate		
Turnover (€ m) / Rating category	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	47.7	55.3	63.1	67.3	72.1	83.9
IV	19.4	19.0	18.2	16.2	13.8	9.9
V–VI	32.9	25.7	18.6	16.5	14.1	6.2

Exhibit 7: **Average total differences of capital requirements in the Basel II IRBA (in percent)**

	Other Retail			Corporate		
Turnover (€ m) / Differences	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Basel II IRBA	-49.3	-50.2	-48.9	-13.3	-10.3	0.0
Estimated	-42.7	-47.4	-39.7	-35.1	-33.9	0.0
Average total difference	**6.6**	**2.8**	**9.2**	**-21.8**	**-23.6**	0.0

Exhibit 7 shows that the average total differences for the corporate portfolio and the retail portfolio move in different directions. For the retail portfolio, the gap between both relative differences from the benchmark is positive but below ten percentage points. We define average total differences below this threshold as economically insignificant. In contrast, for all SME loans assigned to the corporate portfolio, the capital requirements based on estimated asset correlations show significantly higher negative differences than Basel II does, with a gap of about 22–24 percentage points.

These results for the IRBA indicate that there is potential for increasing the relative distance between the capital requirements for SME loans in the corporate portfolio and the capital requirements for loans to larger corporates. This could be achieved, for example, by providing a capital relief for lowering the risk weights of SMEs relative to their current treatment for a certain turnover class only, or by adjusting the asset correlation parameters of the IRBA formula. In general, the results of Exhibit 7 are not surprising and confirm a perception already discussed when the Basel II framework was designed, namely that splitting the SMEs between the corporate portfolio and the retail portfolio gave rise to a cliff effect between the two portfolios.

Evaluation of Capital Requirements in the Standardized Approach

We compare the relative level of capital requirements implied by the asset correlation estimates with the RSA capital requirements. Exhibit 8 corresponds with Exhibit 7 for the IRBA but calculates the average total differences in the RSA. The RSA risk weight function is simply a step function with a risk weight of 100% if the firm is treated as a corporate exposure and 75% if it is assigned to the retail portfolio, i.e. if the exposure to the borrower does not exceed € 1 million, which is comparable with a turnover of up to € 2.5 million.

Exhibit 8: **Average total differences of capital requirements in the RSA (in percent)**

	Other Retail			Corporate		
Differences \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Basel II RSA	-25.0	-25.0	-25.0	0.0	0.0	0.0
Estimated	-42.7	-47.4	-39.7	-35.1	-33.9	0.0
Average total difference	**-17.7**	**-22.4**	**-14.7**	**-35.1**	**-33.9**	0.0

The results for the RSA are considerably stronger and economically more significant than those for the IRBA. The estimated capital requirements differ to a much greater extent from the benchmark "large corporates" (-34% up to -47%) than the regulatory figures (0% up to -25%). For SMEs in the corporate portfolio, the results are directionally in line with those for the IRBA, but the average total differences are higher up to a level of 35 percentage points. In comparison to the corporate portfolio, the empirical results for the SME loans in the retail portfolio indicate a lower but economically significant capital relief potential between 15 and 22 percentage points. To sum up, for all loans assigned to the SME portfolio, the empirical results suggest that the relative reduction compared to large firms is significantly higher than reflected in the current capital requirements. Before these results are interpreted in terms of a policy message, it needs to be considered that the RSA is less risk-sensitive than the IRB approach in general, which justifies a more conservative calibration.

Policy Message

In our paper we have identified two cases in which our empirical results suggest that the relative differences between the capital requirements for large corporates and those for SMEs (in other words, the capital relief for SMEs) are lower in the current regulatory framework than suggested by our empirically estimated asset correlations. Since these relative differences reflect the regulatory capital relief granted for SMEs they may – in certain cases and if taken at face value – indicate a potential for an even higher capital relief. This would be equivalent to lowering the regulatory capital requirements for SMEs, for instance by lowering the asset correlation values in the IRBA formula or by lowering the RSA risk weights directly. Before drawing this inference as the policy message of this paper, the following important caveats need to be carefully considered:

The RSA was deliberately calibrated more conservatively than the IRB approaches. This can be explained by the significantly lower risk sensitivity of the RSA and the regulatory intention to retain incentives in terms of a *ceteris paribus* capital relief when banks switch from the RSA to the more risk-sensitive IRB approach. The more conservative calibration is one reason why the capital requirements in the RSA are currently independent of firm size, which is one important driver for the empirically observed lower potential for reductions of the capital requirements. It also suggests that at least a substantial part of the 15%–35% difference between the current capital relief in the RSA and the capital relief implied by our new empirical results can be explained by this original calibration target.

Although the time series of default rates is longer than in earlier studies on the calibration of Basel II, it is still relatively short, and the use of semi-annual rather than annual time intervals for measuring the default rates does not counterbalance the limitation that the development of the German economy is only captured over seven years. The substantial noise in asset correlation estimates from such short time series has been well documented, for example in Düllmann et al. [2010] by Monte Carlo simulations. Furthermore, although the sample period covers the recent global financial crisis, the German SME sector appears to have been surprisingly unaffected.

Since the regulatory minimum capital requirements are harmonized internationally nowadays, their modification appears reasonable only if the results of this study are also broadly representative for other countries. This applies all the more since the development of the German economy, for example, has differed positively from that in other European countries during the financial crisis. Therefore, further analyses would appear to be useful, especially for countries which also have a strong SME business sector.

Finally, since any adjustment of the relative calibration of risk weights for different asset categories will affect the overall level of capital in the banking system, one can argue that such an adjustment would also require a modification of the overall (or level) calibration to ensure that it is neutral to the overall level of capital.

A Appendix

Estimation Methodology

ML Estimator

The ML estimator proposed by Gordy and Heitfield [2002] draws on the fact that the number of defaults D in a (homogeneous) obligor bucket with n obligors follows a binomial distribution in each period, conditional on systematic factor X. The default probability conditional on $X = x$ is defined as

$$P(D = d|X = x) = \binom{n}{d} g(x; \rho, \gamma)^d \left(1 - g(x; \rho, \gamma)\right)^{n-d}.$$

The ML estimator of ρ is determined numerically by maximizing the log-likelihood function

$$LL(a, b; \rho, \gamma) = \sum_t \log\left(L_t(a_t, b_t; \rho, \gamma)\right),$$

where a_t denotes the $(T \times 1)$ vector of the total number of obligors for T time periods, b_t the $(T \times 1)$ vector for the number of defaulted obligors and

$$L_t(a_t, b_t; \rho, \gamma) = \int_{\mathbb{R}} \binom{a_t}{b_t} g\left(\Phi^{-1}(x); \rho, \gamma\right)^{b_t} \left(1 - g(\Phi^{-1}(x); \rho, \gamma)\right)^{a_t - b_t} \varphi(x) dx$$

with φ representing the probability density function of the standard normal distribution.

Method-of-Moments (MM) Estimator

The (asymptotic) MM estimator matches the first and second moments of the conditional default probability with the first and second moment of the observable default rates. The first moment is estimated by the average default rate, given by

$$E[g(x)] = \bar{p},$$

with $g(x)$ denoting the default probability conditional on $X = x$, and the second moment is estimated by the sample variance of the default rate, given by

$$Var[g(x)^2] = \Phi_2\left(\Phi^{-1}(\bar{p}), \Phi^{-1}(\bar{p}), \rho\right) - \bar{p}^2,$$

where $\Phi_2(\cdot)$ is the cumulative bivariate Gaussian distribution function.

B Appendix

Results for Semi-Annual Data, MM Estimator

Exhibit B.1: **MM estimates for asset correlations and probabilities of default (in percent)**

Asset correlation estimates						
Rating category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	0.69	0.78	0.89	0.95	1.09	2.10
	(0.33)	(0.35)	(0.40)	(0.45)	(0.49)	(0.96)
IV	0.66	0.56	0.91	1.14	1.05	2.45
	(0.32)	(0.26)	(0.41)	(0.54)	(0.48)	(1.23)
V–VI	0.57	0.36	0.60	0.82	1.00	2.19
	(0.23)	(0.15)	(0.25)	(0.36)	(0.41)	(1.06)

PD estimates (one-year horizon)						
Rating category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	0.66	0.56	0.56	0.56	0.49	0.41
	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)	(0.03)
IV	2.10	2.32	2.54	2.70	2.48	2.53
	(0.07)	(0.07)	(0.09)	(0.11)	(0.10)	(0.16)
V–VI	10.07	10.52	11.31	10.70	9.72	8.84
	(0.22)	(0.18)	(0.25)	(0.28)	(0.28)	(0.43)

Standard errors determined by bootstrapping are given below in brackets.

Exhibit B.2: **Average total differences of capital requirements in the Basel II IRBA based on MM estimates (in percent)**

	Other Retail			Corporate		
Differences \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Basel II IRBA	-49.0	-49.9	-48.6	-12.7	-9.9	0.0
Estimated	-45.2	-48.1	-36.8	-30.9	-31.7	0.0
Average total difference	3.8	1.9	11.8	-18.1	-21.8	0.0

Exhibit B.3: **Average total differences of capital requirements in the Basel II RSA based on MM estimates (in percent)**

	Other Retail			Corporate		
Differences \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Basel II RSA	-25.0	-25.0	-25.0	0.0	0.0	0.0
Estimated	-45.2	-48.1	-36.8	-30.9	-31.7	0.0
Average total difference	-20.2	-23.1	-11.8	-30.9	-31.7	0.0

C Appendix

Results for Yearly Data, ML Estimator

Exhibit C.1: **ML estimates for asset correlations and probabilities of default, yearly data (in percent)**

Asset correlation estimates						
Rating category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	0.21	0.64	0.71	0.67	0.96	1.79
	(0.17)	(0.40)	(0.44)	(0.44)	(0.58)	(1.13)
IV	0.28	0.45	0.76	0.84	0.79	1.72
	(0.22)	(0.29)	(0.48)	(0.56)	(0.51)	(1.18)
V–VI	1.72	1.72	1.72	0.96	1.25	3.33
	(1.18)	(1.18)	(1.18)	(0.59)	(0.73)	(2.08)
PD estimates (one-year horizon)						
Rating category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	1.32	1.17	1.13	1.15	0.10	0.92
	(0.08)	(0.11)	(0.12)	(0.12)	(0.12)	(0.16)
IV	4.26	4.66	5.16	5.60	5.13	5.41
	(0.25)	(0.32)	(0.45)	(0.52)	(0.46)	(0.75)
V–VI	22.98	20.88	19.84	21.52	19.36	18.16
	(1.22)	(0.82)	(0.78)	(1.42)	(1.49)	(2.42)

Standard errors determined analytically from asymptotic Fisher information matrices are given below in brackets.

Exhibit C.2: **Average total differences of capital requirements in the Basel II IRBA, yearly data (in percent)**

	Other Retail			Corporate		
Differences \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Basel II IRBA	49.6	50.5	50.2	-17.2	-13.4	0.0
Estimated	53.2	40.1	36.5	-39.1	-33.1	0.0
Average total difference	3.5	10.4	13.8	-21.9	-19.8	0.0

Exhibit C.3: **Average total differences of capital requirements in the Basel II RSA, yearly data (in percent)**

	Other Retail			Corporate		
Differences \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Basel II RSA	-25.0	-25.0	-25.0	-0.0	-0.0	-0.0
Estimated	53.2	40.1	36.5	-39.1	-33.1	0.0
Average total difference	-28.2	-15.1	-11.5	-39.1	-33.4	0.0

D Appendix

Results for Yearly Data, MM Estimator

Exhibit D.1: **MM estimates for asset correlations and probabilities of default, yearly data (in percent)**

Asset correlation estimates						
Rating category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	0.35	0.86	0.95	0.89	1.21	2.34
	(0.24)	(0.51)	(0.57)	(0.56)	(0.70)	(1.33)
IV	0.45	0.54	1.11	1.19	1.16	2.46
	(0.30)	(0.34)	(0.67)	(0.74)	(0.73)	(1.16)
V–VI	0.53	0.33	0.60	0.53	1.20	2.66
	(0.31)	(0.20)	(0.36)	(0.34)	(0.72)	(1.65)
PD estimates (one-year horizon)						
Rating category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I-III	0.66	0.56	0.56	0.56	0.49	0.41
	(0.04)	(0.06)	(0.06)	(0.06)	(0.06)	(0.08)
IV	2.12	2.35	2.58	2.74	2.51	2.57
	(0.14)	(0.16)	(0.25)	(0.27)	(0.25)	(0.39)
V–VI	10.61	11.11	12.00	10.61	10.22	9.26
	(0.52)	(0.42)	(0.61)	(0.53)	(0.75)	(1.09)

Standard errors determined by bootstrapping are given below in brackets.

Exhibit D.2: **Average total differences of capital requirements in the Basel II IRBA based on MM estimates, yearly data (in percent)**

	Other Retail			Corporate		
Differences \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Basel II IRBA	-49.0	-50.0	-48.5	-13.0	9.9	0.0
Estimated	-61.6	-50.3	-38.3	-40.0	-31.8	0.0
Average total difference	-12.6	-0.4	10.3	-27.0	-22.0	0.0

Exhibit D.3: **Average total differences of capital requirements in the Basel II RSA based on MM estimates, yearly data (in percent)**

	Other Retail			Corporate		
Differences \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Basel II RSA	-25.0	-25.0	-25.0	0.0	0.0	0.0
Estimated	-61.6	-50.3	-38.3	-40.0	-31.8	0.0
Average total difference	-36.6	-25.3	-13.3	-40.0	-31.8	0.0

Endnotes

[1]This treatment has been maintained without change in the Basel III framework.

[2]In the IRBA the capital requirements are computed by multiplying the credit exposure by a risk weight that is a function of the default probability, the recovery rate, the maturity and the asset type of the loan. In the RSA the risk weight is tabulated and depends both on the borrower type and an external rating, i.e. a rating given by an acknowledged rating agency. Very often in this paper the terms "capital requirement" and "risk weight" can be used interchangeably.

[3]The availability of "number of borrowers" and "number of defaults" per bucket allows us to merge buckets quite flexibly if the number of observations becomes too low for robust estimation results.

[4]In addition we also employ the Asymptotic Maximum Likelihood Estimator that has been analysed by Düllmann et al. [2010] in small samples. The results for this estimator are available from the authors on request.

[5]This segment comprises firms with a yearly turnover of at least €50 million; the size adjustment in the IRBA risk weight function is zero for this segment.

References

Basel Committee on Banking Supervision. International convergence of capital measurement and capital standards – a revised framework comprehensive version. 2006.

Tobias Berg, Bernhard Gehra, and Michael Kunisch. A certification model for regulatory arbitrage: Will regulatory arbitrage persist under Basel III? *The Journal of Fixed Income*, 21(2):39–56, 2011.

Michel Dietsch and Joel Petey. Should SME exposures be treated as retail or corporate exposures? A comparative analysis of default probabilities and asset correlations in French and German SMEs. *Journal of Banking & Finance*, 28(4):773–788, 2004.

Klaus Düllmann and Harald Scheule. Determinants of the asset correlations of German corporations and implications for regulatory capital. Unpublished Working Paper, 2006.

Klaus Düllmann, Jonathan Küll, and Michael Kunisch. Estimating asset correlations from stock prices or default rates: which method is superior? *Journal of Economic Dynamics & Control*, 34(2010):2341–2357, 2010.

Michael Gordy and Erik Heitfield. Estimating factor loadings when ratings performance data are scarce. Working Paper, Federal Reserve Board, 2000.

Michael Gordy and Erik Heitfield. Estimating default correlations from short panels of credit rating performance data. Working Paper, Federal Reserve Board, 2002.

Michael B. Gordy. A risk-factor model foundation for ratings-based bank capital rules. *Journal of Financial Intermediation*, 12(3):199–232, 2003.

Lutz Hahnenstein. Calibrating the creditmetrics correlation concept - empirical evidence from Germany. *Financial Markets and Portfolio Management*, 18(4):358–381, 2004.

Joint Banking Initiative for the Financial Location of Germany. Initiative Finanzstandort Deutschland (IFD) – Rating Brochure, 2010.

Steve Mercieca, Klaus Schaeck, and Simon Wolfe. Bank market structure, competition, and SME financing relationships in European regions. *Journal of Financial Services Research*, 36: 137–155, 2009.