Estimating the distribution of total default losses on the Spanish Financial system

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Presentation Outline

Section 1

Introduction and motivation

- **Credit risk** refers to the risk that a borrower will **default** on any type of debt by failing to make **required payments**.
- The credit risk is the most relevant type of risk that **financial institutions** face. In fact over the financial crisis, the credit losses have been the main source of P&L losses in the financial system.
- Several financial institutions **defaulted** or had to be **rescued** over the financial crisis. In the case of Spain the most relevant ones were: **Bankia, CAM, CatalunyaCaixa** and **NCG**.
- Given the negative impact of the default of a financial institution on the financial system it is **crucial** having accurate tools to **measure and allocate** the credit risk of the financial system.
- The most extended credit risk measurement model is the **Vasicek (1987) model**:
  - This is a quite **flexible model** that under restrictive conditions (**ASRF**) allows for a quick measurement of the credit risk.
  - Under **general conditions** the estimation-allocation of the risk can be very **time consuming**.
  - The model assumes **constant recoveries vs random recoveries or market valuation**.

Contributions of the paper

- We show how to estimate the Vasicek model inputs (PD, LGD, EAD, α) and use the importance sampling methods **to measure the risk of the Spanish financial system.**
- We **extend the IS framework** introduced in Glasserman and Li (2005) to deal with **random recoveries** and **market mode** valuation.
- According to our results the portfolio **loss distribution and risk allocation vary considerably** depending on the model considered, constant vs random LGD, default vs market mode, and the risk allocation rule, VaR vs ES.
- These methods can be very **useful for regulators to** identify and measure the impact of the **systemically important financial institutions or to stress test** the financial system.

Section 2

Credit Risk

- The **default of a counterparty** j is driven by its assets value V_j:

$$V_j = r_j Y_j + \sqrt{1 - r_j^2}\varepsilon_j = \sum_{f=1}^{k} \alpha_{f,j} z_f + \varepsilon_j \sqrt{1 - \sum_{f=1}^{k} \alpha_{f,j}^2}$$

$z_f \equiv$ macroeconomic common factors, $\varepsilon_j \equiv$ idiosyncratic risk factors.

- **Default happens if** $V_j < k_j = \Phi^{-1}(PD_{C,j})$, where $PD_{C,j}$ is the average historical default rate.
- **Total portfolio losses:**

$$L = \sum_{j=1}^{M} x_j = \sum_{j=1}^{M} EAD_j LGD_j \mathbf{1}(V_j \leq \Phi^{-1}(PD_{j,C}))$$

  - EAD_j is the amount owed in the default moment.
  - LGD_j = "1 - final recovery rate".

- **Particular case:** The **ASRF** model is the Vasicek model with a single macroeconomic risk factor and many identical clients.
- **Closed-form expression for the loss** for a given probability level:

$$L(q) = EAD \times LGD \times \Phi\left(\frac{\Phi^{-1}(PD) - \alpha\Phi^{-1}(1-q)}{\sqrt{1-\alpha^2}}\right)$$

- Regulatory capital requirements (BASEL - BIS) are based on this model.
- Under **non-ASRF** portfolios, we can use Monte Carlo (MC) simulations and approximate methods. Among those methods we have the **importance sampling**, the **saddlepoint methods** and the **Taylor expansion based methods**.
- **Risk allocation**, based on the VaR/ES (mathematical) derivatives. The two most extended methods to allocate a risk value of l are:
  - **Value at Risk** (VaR) based: $CVaR_j = E\left(x_j \;\middle|\; \sum_{j=1}^{M} x_j = l\right)$.
  - **Expected Shortfall** (ES) based: $CES_j = E\left(x_j \;\middle|\; \sum_{j=1}^{M} x_j \geq l\right)$.

Section 3

Spanish Portfolio

Spanish Portfolio

- We analyze the portfolio of **Spanish financial institutions** covered by the deposit guarantee fund at Dec 2010.
- **EAD**: Balance sheet information at this time. We sum the information of those institutions belonging to the same group.
- **PD**: Based on the S&P, Moody's and Fitch external ratings.
  - Institutions with **no external rating** available are assigned one notch less than the average rating of the portfolio with external rating.
  - We **calibrate** a long-term PD curve to the empirical average default data (period 1980-2009) imposing that a AA- has a PD of 3 bps.
- **LGD**: We update the results obtained in Bennet (2002) for the period 1986-2009 using FDIC public data and extend them to **losses on total assets** rather than losses to FDIC. [1]

[1] We assume that the Spanish closure procedures are similar to those in USA.

Spanish Portfolio

Table: Financial institutions involved in a merger / acquisition or belonging to the same corporation at December, 2010.

New Entity	Original Institutions
Banca Civica	Caja Municipal de Burgos, Caja Navarra, Caja Canarias, CajaSol, Caja Guadalajara
Banco Base	Caja Asturias, Banco de Castilla La Mancha, Caja Cantabria, Caja Extremadura
Banco Mare Nostrum	Caja Murcia, Caixa Penedés, Caja Granada, Caja Sa Nostra
Banco Popular	Banco Popular, Banco Popular Hipotecario, Banco Popular-e, Popular banca privada
Bankia	Caja Madrid, Bancaja, Caixa Laietana, Caja Avila, Caja Segovia, Caja Rioja, Caja Insular
BBK	BBK, Cajasur
BBVA	BBVA, Finanzia, Banco Depositario BBVA, UNO-E Bank
Caixabank	La Caixa, Caixa Girona, Microbank
Caja 3	Caja Inmaculada, Caja Burgos CCO, Caja Badajoz
Caja España de Inversiones	Caja España, Caja Duero
CatalunyaCaixa	Caixa Cataluña, Caixa Tarragona, Caixa Manresa
Novacaixagalicia	Caja Galicia, Caixanova
Santander	Banco Santander, Banesto, Santander Investment, Openbank, Banif, Santander Consumer Finance
Unicaja	Unicaja, Caja Jaén
Unnim	Caixa Sabadell, Caixa Terrassa, Caixa Manlleu

Spanish Portfolio

Table: Calibrated default probabilities

S&P	Calibrated PD
AAA	0.0249%
AA+	0.0253%
AA	0.027%
AA-	0.03%
A+	0.0356%
A	0.0459%
A-	0.0651%
BBB+	0.1005%
BBB	0.1659%
BBB-	0.2871%
BB+	0.5112%
BB	0.9257%
BB-	1.6925%
B+	3.111%
B	5.735%
B-	10.5892%

Table: LGD on deposits and on assets for the period 1986-2009.

Assets	Count	Mean (deposits)	Mean (assets)
$<$ \$1bn	1148	18.61%	25.65%
\$1bn - \$5bn	49	15.50%	21.37%
\$5bn - \$15bn	7	9.95%	13.72%
$>$ \$15bn	8	6.39%	8.82%

Spanish Portfolio

Table: Top 25 institutions risk parameters used to obtain the portfolio loss distribution

Institution	PD	LGD	EAD (MM €)	EAD Share	r_j
SANTANDER	0.0272%	8.8%	602,697	20.6%	54.6%
BBVA	0.029%	8.8%	402,941	13.8%	54.6%
BANKIA	0.0878%	8.8%	328,277	11.2%	54.2%
LA CAIXA	0.0371%	8.8%	286,046	9.8%	54.5%
POPULAR	0.0619%	8.8%	134,793	4.6%	54.4%
SABADELL	0.0616%	8.8%	86,559	3%	54.4%
CATALUNYACAIXA	0.5025%	8.8%	76,585	2.6%	51.6%
NCG	0.372%	8.8%	73,493	2.5%	52.4%
BANCA CIVICA	0.1106%	8.8%	71,374	2.4%	54%
CAM	0.482%	8.8%	70,667	2.4%	51.8%
MARE NOST.	0.1606%	8.8%	69,859	2.4%	53.7%
BANKINTER, SA	0.0554%	8.8%	55,665	1.9%	54.4%
BASE	0.435%	8.8%	54,504	1.9%	52%
CAJA ESPAA	0.2824%	8.8%	45,711	1.6%	52.9%
BBK	0.4446%	8.8%	45,215	1.5%	52%
IBERCAJA	0.0726%	8.8%	44,989	1.5%	54.3%
UNICAJA	0.055%	8.8%	34,344	1.2%	54.4%
BARCLAYS	0.0245%	8.8%	34,339	1.2%	54.6%
BANCO PASTOR	0.5025%	8.8%	31,134	1.1%	51.6%
UNIMM	0.9257%	8.8%	28,353	1%	49.4%
CAJAMAR	0.1785%	8.8%	28,340	1%	53.6%
BANCO DE VALENCIA	0.4148%	8.8%	23,530	0.8%	52.1%
CAJA LABORAL	0.1635%	8.8%	20,998	0.7%	53.7%
KUTXA	0.0459%	8.8%	20,851	0.7%	54.5%
CAJA 3	0.2303%	8.8%	20,763	0.7%	53.3%

Spanish Portfolio

- $\alpha_{f,j}$: based on **BASEL regulatory parameters** and geographic exposition
  - The factor sensitivity is based on $r_j = \sqrt{1.25}\sqrt{0.12\omega + 0.24\,(1-\omega)}$ where $\omega = \frac{1-e^{-50PD}}{1-e^{-50}}$.
  - All the institutions are exposed to a **Spanish macro factor but BBVA and Santander** that are exposed to more macroeconomic factors according to their net interest income information.
  - Measuring **diversification is crucial** under a multi-factor model.

Table: BBVA and Santander country exposition according to the net interest income data published in their 2010 Annual Reports.

	Spain	Mexico	United States	Argentina	Chile	Colombia	Peru	Venezuela, RB	Portugal	United Kingdom	Brazil	Italy	Finland	Germany
BBVA	37.7%	33.5%	9.6%	2.8%	4.0%	4.0%	4.8%	3.7%	0%	0%	0%	0%	0%	0%
Santander	18.8%	5.9%	6.8%	0%	5.3%	0%	0%	0%	2.6%	14.5%	36.8%	0.7%	0.8%	7.5%

- **Macroeconomic factor correlation**: based on the GDP correlation between countries.

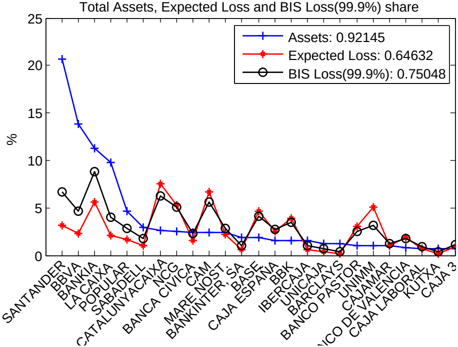


Figure: Assets, Expected Loss, and Basel 99.9% loss share of the top 25 in assets Spanish financial institutions.

- In the **ASRF** model, the 99.9% probability losses are **13,733 MM €**.

Section 4

Importance sampling methods

- **Importance sampling** is a technique to estimate expectations of random variables based on Monte Carlo simulations.
- **Idea:** To estimate the expectation generating simulations with a different distribution and adjust each simulation using a weight.

$$Prob(L \geq l) = E(\mathbf{1}(L \geq l)) \approx \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}(L_i \geq l) \frac{f(L_i)}{g(L_i)}$$

where L_i are sampled from $g(L)$.

- Glasserman and Li (2005) suggest two changes in the simulation process:
  - Change the **default behavior conditional** to the macro scenario, z_f.
  - Change the **distribution of the macroeconomic** factors z_f.
- **Basic idea:** To **increase** the number of simulations generating **high losses** to have a better estimate of the probability.

- **1:** Change the default probability conditional to a macroeconomic scenario:

$$PD_{j,Z,\theta} = \frac{PD_{j,Z} e^{LGD_j EAD_j \theta}}{1 + PD_{j,Z}(e^{LGD_j EAD_j \theta} - 1)}$$

- The **weight** for the simulation i is:

$$W_{1,i} = \prod_{j=1}^{M} \left(\frac{PD_{j,Z}}{PD_{j,Z,\theta}} \right)^{D_{j,i}} \left(\frac{1 - PD_{i,Z}}{1 - PD_{j,Z,\theta}} \right)^{1-D_{j,i}}$$

where $D_{j,i}$ is the default indicator of the client j in the simulation i.

- The parameter θ is set so that:

$$\sum_{j=1}^{M} EAD_j PD_{j,Z,\theta} LGD_j \geq l$$

- **2:** Change the distribution of the macroeconomic factors.
- Sample Z from a normal distribution with the **same mode** as the optimum distribution, $g(Z) \sim N(\mu, I)$:

$$\mu = \max_{Z} \left\{ Prob(L \geq l|Z) e^{-\left(Z'Z\right)/2} \right\}$$

- As $Prob(L \geq l|Z)$ is unknown, these authors propose a **normal approximation** based on $E(L|Z)$ and $E(L^2|Z)$.
- The new **weight** for the simulation i is:

$$W_{2,i} = e^{-\mu' Z + \mu' \mu/2}$$

Section 5

Portfolio Results

Portfolio Results - Loss distribution

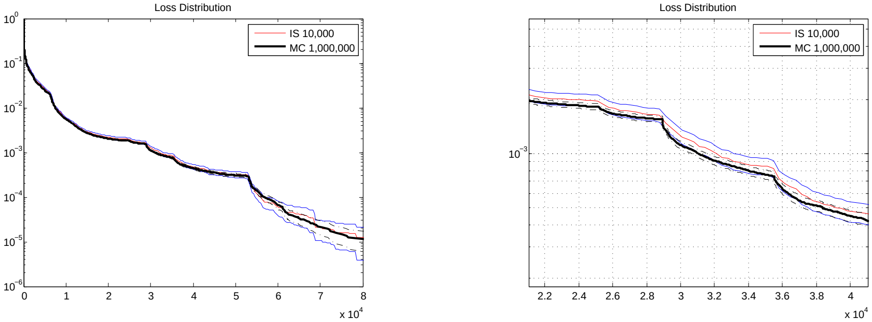


Figure: Loss distribution using 10,000 importance sampling (IS) and 1,000,000 Monte Carlo (MC) simulations.

- The 99.9% probability losses are **32,102 MM €**, 2.3 times more than in the ASRF framework!.
- The confidence intervals with just 10,000 simulations are very narrow.

Portfolio Results - Risk allocation

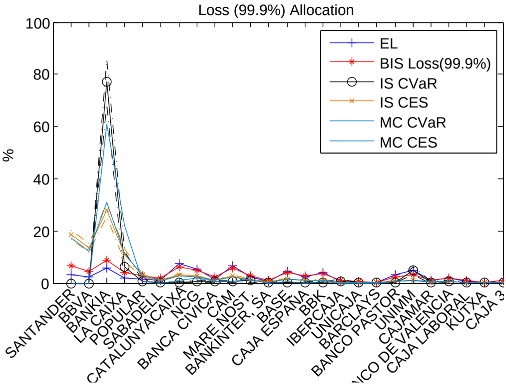
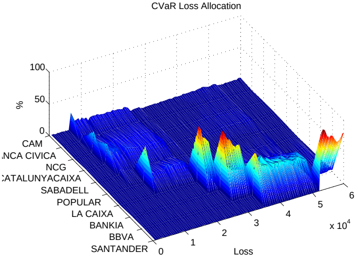


Figure: Risk allocation.

- Santander and BBVA's VaR based **risk allocation is zero** as their LGDs are 53,146 MM € and 35,531 MM €, respectively.

Portfolio Results - Risk allocation



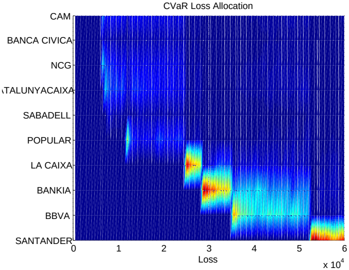


Figure: CVaR risk allocation share for a range of losses

- The VaR based risk allocation has a sharp profile.

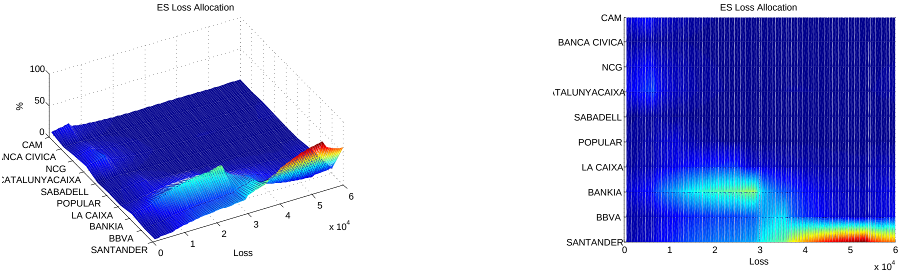


Figure: CES risk allocation share for a range of losses

- The ES based risk allocation is a more smooth allocation rule.

Section 6

Model Extensions

Under random LGD there is a macroeconomic factor affecting the LGD behavior (z_{LGD}). Conditional to this factor, **the LGD can be**

- **Constant:**

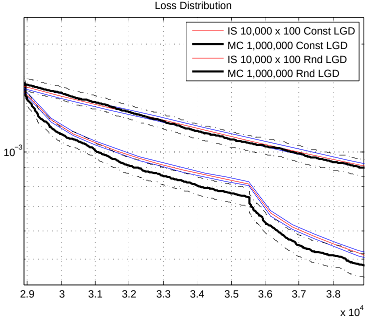
$$LGD_{j,Z} = \Phi\left(\frac{\Phi^{-1}(LGD_{j,C}) - \alpha_j z_{LGD}}{\sqrt{1-\alpha_j^2}}\right)$$

- **Random:**

$$LGD_{j,z,\gamma_j} = \Phi\left(\frac{\Phi^{-1}(LGD_{j,C}) - \alpha_j(r z_{LGD} + s\gamma_j)}{\sqrt{1-\alpha_j^2}}\right)$$

- In both cases, the parameter α_j can be inferred from the evolution of the average *LGD* over the period 1986-2009.
- The parameter s can be inferred from the average variability of the *LGD* within a year.

Constant Conditional LGD (*LGD^C*):



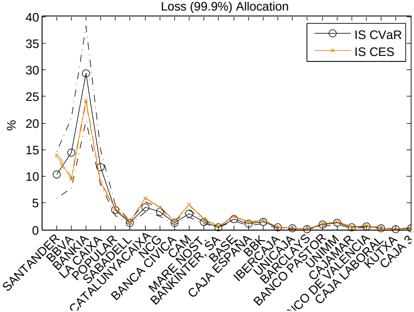
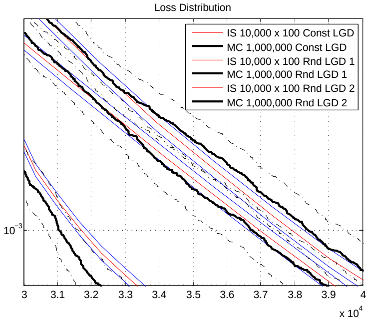


Figure: Comparison of the random LGD (Rnd LGD) and constant LGD (Const LGD) loss distributions.

- 99.9% probability losses ↑ to 36,970 MM €. Now, all the institutions have **some risk allocated but** the confidence intervals get increased.

Random Conditional LGD (*LGD^R*):



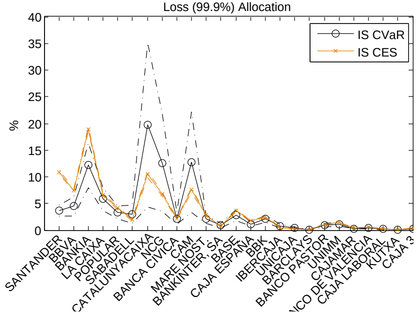


Figure: Comparison of the two random LGD models (Rnd LGD^C / Rnd LGD^R) and constant LGD (Const LGD) loss distributions.

- 99.9% probability losses ↑ to 37,934 MM €. The **risk allocation changes** & bigger confidence intervals (CAM, CAT.CAIXA, NCG).

Model extensions - Random LGD

Table: Comparison of the 99.9% probability loss levels under different random LGD models. We consider a pure macroeconomic LGD (LGD^C), based on transformations of a random normal macroeconomic variable z_{LGD}, the random LGD conditional to the macroeconomic variable z_{LGD} (LGD^R), and the case of $LGD|z_{LGD}$ with Beta and Gamma distributions.

Model	Loss (MM €)	Model	Loss (MM €)
Normal LGD^C	37,160	Probit Normal LGD^C	35,999
Normal LGD^R	38,131	Probit Normal LGD^R	35,318
Log-Normal LGD^C	29,309	Normal^2 LGD^C	36,826
Log-Normal LGD^R	36,139	Normal^2 LGD^R	36,587
Logit Normal LGD^C	35,909	Beta LGD^R	37,616
Logit Normal LGD^R	34,997	Gamma LGD^R	37,578

- **Similar 99.9% probability results** for all the models tested. Results vary between 38,131 MM € and 34,997 MM € for all the models except for the pure macro **Log-Normal** LGD that produces 29,309 MM € losses.

Model extensions - Market mode

- **Losses** are due to **changes in the rating** of the clients (including default). **Losses on current MtM**.
- **Loans have to be valued according to** the rating of the clients:
  - We use average CDS-5Y spread LGD adjusted by rating grade for 2008-2011.
  - We assume an average credit maturity of 3 years.
  - We obtain the discount factors of the different ratings.
- **For the migration rule, we use** the same cycle variability parameter ($\alpha_{f,j}$) as for the default mode and a migration matrix.
- We use the **accumulated migration probabilities** $AccumMP_{j,C,IR,FR}$.
- Example:

$$AccumMP_{j,C,IR,B-} = MP_{j,C,IR,B-} + MP_{j,C,IR,CCC} + MP_{j,C,IR,D}$$

- Then, conditional to z_f, we define

$$AccumMP_{j,Z,IR,FR} = \Phi\left(\frac{\Phi^{-1}(AccumMP_{j,C,IR,FR}) - \sum_{f=1}^{k}\alpha_{f,j}z_f}{\sqrt{1-\sum_{f=1}^{k}\alpha_{f,j}^2}}\right)$$

- From $AccumMP_{j,Z,IR,FR}$ we recover $MP_{j,Z,IR,FR}$.

Model extensions - Market mode

Table: Average 1-year rating migration matrix from S&P (2010). Period (1981-2009)

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	91%	4%	3%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0.0249%
AA+	2%	79%	12%	4%	1%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0.0253%
AA	1%	1%	84%	8%	3%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0.027%
AA-	0%	0%	5%	80%	10%	3%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0.03%
A+	0%	0%	1%	5%	81%	9%	3%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0.0356%
A	0%	0%	0%	1%	5%	81%	7%	3%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0.0459%
A-	0%	0%	0%	0%	1%	7%	79%	8%	3%	1%	0%	0%	0%	0%	0%	0%	0%	0.0651%
BBB+	0%	0%	0%	0%	0%	1%	7%	78%	9%	2%	0%	0%	0%	0%	0%	0%	0%	0.1005%
BBB	0%	0%	0%	0%	0%	1%	1%	7%	80%	6%	2%	1%	0%	0%	0%	0%	0%	0.1659%
BBB-	0%	0%	0%	0%	0%	0%	0%	2%	9%	76%	6%	3%	1%	1%	0%	0%	0%	0.2871%
BB+	0%	0%	0%	0%	0%	0%	0%	1%	2%	13%	69%	7%	4%	1%	1%	0%	1%	0.5112%
BB	0%	0%	0%	0%	0%	0%	0%	0%	1%	3%	9%	71%	9%	3%	2%	1%	1%	0.9257%
BB-	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	2%	9%	71%	9%	4%	1%	1%	1.6925%
B+	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	2%	7%	73%	9%	3%	2%	3.111%
B	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	2%	9%	66%	9%	7%	5.735%
B-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	3%	10%	60%	14%	10.5892%
CCC/C	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	2%	3%	11%	63%	19.5689%

Table: Discount factor by rating grade based on the average CDS spread and 3 years average maturity.

Investment Grade		Speculative Grade	
Rating	Discount Factor	Rating	Discount Factor
AAA	98.71%	BB+	95.79%
AA+	98.69%	BB	95.51%
AA	98.6%	BB-	94.99%
AA-	98.37%	B+	94.04%
A+	97.93%	B	92.33%
A	97.14%	B-	89.26%
A-	96.96%	CCC/C	83.93%
BBB+	96.63%		
BBB	96.02%		
BBB-	95.94%		

Model extensions - Market mode

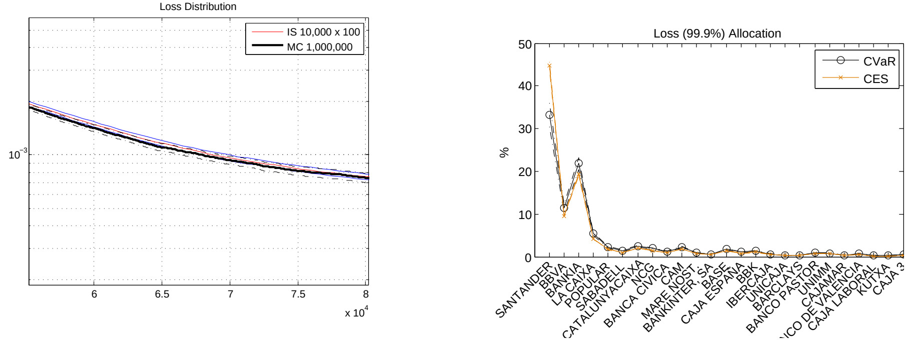


Figure: Market mode loss distribution.

- 99.9% probability losses are **68,852 MM €** (additional to the current MtM loss of 79,006 MM €) and the risk allocated to Santander and BBVA increases.

Section 7

Conclusions

Conclusions:

- We have successfully **extended the IS framework** introduced in Glasserman and Li (2005) to deal with **random recoveries** and **market mode** valuation.
- These methods have been used **to measure the risk of the Spanish financial system.**
- Portfolio **results vary considerably depending on the model** considered, VaR vs ES, constant vs random LGD, default vs market mode.
- These methods can be very **useful for regulators to** identify and measure the impact of the **systemically important financial institutions or to stress test** the financial system.

Further Research:

- Use **point in time PDs**.
- **Structural** approach for the portfolios **LGD**.
- Improve the **interrelation measurement**. Factor diversification. Contagion models.

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