Stress Testing the Credit Risk of Mortgage Loans:
The Relationship between Portfolio-LGD and the Loan-to-Value Distribution

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Abstract:

In this paper, we analyze the impact of stressed recovery rates for collateral on the average loss given default (LGD) in portfolios of mortgage loans. We demonstrate that the extent of additional losses under stress depends heavily on the shape of the portfolio's loan-to-value (LTV-) distribution. Hence, a comparison of stressed mean portfolio LGDs across banks that does not properly account for differing LTV-distributions will generally not lead to meaningful conclusions. Furthermore, we derive a closed-form solution for the mean portfolio LGD under the assumption of beta-distributed LTV-ratios and we check the robustness of this approximation for several hypothetical bank portfolios. The formula seems a meaningful starting point for benchmarking analyses by risk managers, rating agencies and regulators with respect to LGDs for mortgage loans.

Keywords: credit risk, loan-to-value (LTV), loss given default (LGD), mortgage loan, recovery rate, stress testing

JEL-Classification: G21, G28, G33, C16, C58

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Contents

1   Introduction

Since the last decade´s great financial crisis, both risk managers in banks and banking regulators have been paying much more attention to stress testing in order to enhance the forward-looking assessment of risk.[1] During the last years, stress testing has become an integral part of banks' internal risk management framework and, furthermore, several supervisory stress testing exercises were conducted, notably by the *European Banking Authority (EBA)* in the European Union as well as by the *Federal Reserve Board (FED)* in the United States of America.[2]

The stress testing approach to credit risk in the banking book is usually[3] built around the two default risk parameters probability of default (PD) for a one-year risk horizon and loss given default (LGD), which also form the main building blocks of the Basel II regulatory capital standard[4] for credit risk. The usual aim of stress testing credit risk is to provide an estimate of the bank's expected loan losses, the "P&L effect", and its expected regulatory capital charge given a certain adverse economic scenario.[5]

Apart from the intensified use of stress testing, regulators within the European Union follow a second path in order to strengthen the resilience of the banking sector and to prevent future crises stemming from the real estate markets. In accordance with the recommendations of the *European Systemic Risk Board (ESRB)* and supported by the so-called *Liikanen*-Report[6], nationally harmonized caps on Loan-to-Value-ratios (LTV-ratios) and Loan-to-Income-ratios (LTI-ratios) are about to be introduced as new macro-prudential instruments. The reasoning reads as follows: "…, lower LTV limits can increase the resilience of the banking system via a lower loss given default, while lower LTI limits can reduce the probability of default."[7]

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[1] See e.g. BCBS (2009).
[2] See http://www.eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing and
http://www.federalreserve.gov/bankinforeg/stress-tests-capital-planning.htm for further details on results and methodology of last years' exercises. An overview about the state-of-the-art in regulatory stress testing can be find in Schuermann (2014).
[3] See e.g. EBA (2011) or Gundlach (2011), especially p. 353. The third common risk parameter exposure at default (EAD) is relevant only for non-deterministic exposures that involve the modeling of either credit conversion factors (CCFs), e.g. for undrawn lines of credit, or of market risk factors, e.g. for derivatives. It is of minor importance for modelling the P&L impact of a mortgage loan portfolio under stress. Tobback et. al. (2014) try to forecast the LGD for revolving home equity lines of credit (HELOCs) based on a US bank's monthly data for 17.346 loans that defaulted between 2002 and 2008 and using both macroeconomic variables and account data including Loan-to-value.
[4] Cf. BCBS (2005).
[5] A few papers attempt to go beyond stressing PDs and LGDs by additionally addressing the issue of correlation modelling in order to assess the impact of stress on credit value at risk or expected shortfall. See e.g. Bonti et. al (2006), Düllmann/Erdelmeier (2011) and Düllmann/Kick (2012).
[6] Cf. HLEG (2012), p. 81.
[7] Cf. ESRB (2013), p. 13.

In this paper, we analyze the relationship between the loss given default (LGD) parameter and the distribution of LTV-ratios in a portfolio of loans that are collateralized by real estate assets (residential or commercial real estate financing) for the scenario of a hypothetical property price shock. The distribution of LTV-ratios across the portfolio under consideration contains all the information about potential under- and overcollateralization that is necessary and sufficient to calculate stressed LGDs on a loan level basis. As we will set forth in this paper, the required modelling, which involves a proper treatment of the bank's LTV-distribution, is rather straightforward, if a full numerical LGD calculation on a loan-level basis can be carried out. Because, however, such a calculation is not feasible for regulators and rating agencies due to the data restrictions they face, the comparison of stressed average LGDs across different portfolios is not trivial and even may give rise to conclusions, which are heavily flawed. In order to nonetheless enable a suitable kind of benchmarking, we derive a closed-form solution for the exposure-weighted, mean portfolio LGD as a function of the recovery rate on collateral under the assumption of beta-distributed LTV-ratios. Finally, we compare the results of this formula for six hypothetical "real world" portfolios with an exact calculation that requires the use of loan-level data. The comparison indicates that the formula may be successfully used as an appropriate "rule-of-thumb", when an assessment of the approximate effect of recovery rate stress on a bank's P&L is needed, but a full numerical LGD calculation on a loan-level basis is not feasible.

Our findings are important for credit risk practitioners who are responsible for the design and the validation of credit risk stress tests. Moreover, regulators, who run benchmarking exercises with stressed and unstressed LGDs for different mortgage loan portfolios across international banks, should be aware of these effects. Finally, the rating agencies, who face a similar methodological challenge when they try to assess the credit risk in covered bonds, could make use of our approach as well.

2   Related literature

In this section, we give a brief overview of the related literature on LGD estimation. Driven by the adoption of the Basel II capital standard in 2005, research focusing on the LGD-parameter in general has significantly increased in recent years. The academic literature is dominated by empirical papers, whose main aim is to identify explanatory variables for the observed LGD variation with statistical methods.[8] Examples are *Dermine/de Carvalho* (2006) and *Grunert/Weber* (2009), who both study the determinants of corporate loan LGDs for a major Portuguese resp. German bank. A further example of this main stream of literature that is more closely related to our case is *Leow/Mues* (2011), who develop a number of statistical models for estimating the LGD of mortgage loans using the recovery database of a large UK bank and who discuss several sets of explanatory variables including the loan-to-value ratio at the time of default, the loan-to-value ratio at the time of origination and various other collateral-related variables. In all the three papers mentioned, the recovery data analyzed consist of historical single-loan LGDs, which were provided each by a single major European bank.

There are another three[9] papers on LGD that explicitly focus on the role of the loan-to-value-ratio for the LGD of mortgage loans and that make use of broader datasets available for the US. *Calem/LaCour-Little* (2004) deploy data from the *Office of Federal Housing Enterprise Oversight* (OFHEO) that contain about 120.000 fixed-rate mortgages which terminated in foreclosure in the 1990ies. The percentage net recovery is estimated via regression as a (non-linear) function of both the current loan-to-value ratio at foreclosure date and the original loan-to-value ratio. Together with another five statistically significant regressor variables (e.g. mortgage age and size), a total of 25% of the sum of squared deviations from the mean is explained by the regression equation. *Qui/Yang* (2009) analyze 241.293 residential mortgage loans that had insurance protection by one of the six private mortgage insurance companies operating in the US and that were settled between 1990 and 2003. Based on a complete workout data set, the authors are able to implement the precise Basel II LGD definition by discounting all cash flows to the time of default. Further, since a complete housing market cycle is covered, *Qui/Yang* (2009) claim to be the first paper that

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[8]   One of the rare exceptions is Jokivuolle/Peura (2003), who study the role of collateral value for LGD on the basis of an option-theoretic model. A more recent analytical approach to LGD estimation is Frye (2014).

[9]   An elder paper that also studies a type of LGD-LTV-relationship in the US is Lekkas et. al. (1993). However, since the term LGD did not exist at that time, when Basel II was not yet looming, the title refers to „loss severity" instead. Losses are calculated simply as the difference between the mortgage balance and the property value excluding any costs. Moreover, sales proceeds are not discounted to the time of default. One of the paper's main results is that defaulted loans with higher initial Loan-to-Value-Ratios will have higher loss severities. For the special case of mortgage related revolving lines of credit (HELOCs) see again Tobback et. al. (2014) referenced in FN 3.

empirically models an economic downturn LGD for residential mortgages. They find that the current loan-to-value ratio, loan-to-value at origination and a downturn indicator (dummy variable) measuring regional house price decline can jointly with seven other regressor variables explain as much as 61% of variation in loss severity. Moreover, they find that current LTV is the single most important determinant of LGD and also a much better predictor for LGD than original LTV. Using only information that is available at the time of origination, *Zhang/Ji/Liu* (2010) study the predictive power of local house price history and house price volatility for the average LGD in their sample. Their sample of 838.683 defaulted subprime residential mortgage loans, which were used as underlying claims for the issuance of asset-backed securities (ABS) by US banks between 1998 and 2008, covers the entire country. The authors find that housing market cycle information can crucially increase the explained variation of LGD. Therefore, their LGD function, which produces high LGD estimates when house prices are high, because they have appreciated over several past periods, and when current loan-to-value ratios are low, may be used as a part of the regulators' countercyclical macroprudential toolkit that aims at reducing the procyclicality of the Basel II framework. Of course, such an approach depends crucially on the ability to produce reliable forecasts for future housing prices.

While the aforementioned related empirical papers try to explain the cross-sectional variation and the time-series behavior in historical loan-level LGD data, this paper addresses a different question:

> *How can the impact of a pre-specified drop in collateral values on average portfolio LGD be properly assessed for banks' portfolios of mortgage loans that differ in their loan-to-value distributions?*

In the current stress testing exercises run by EU or US regulating authorities such pre-specified declines in collateral value are typically given in the form of stressed macro-economic variables, e.g. for commercial or residential property prices on a countrywide basis.[10] The scenarios have to be applied to all participating banks' portfolios for a certain reference date. Since the transmission of macro-economic stress into credit risk parameters is of vital importance for the stresstests' results (i.e. the stressed capital ratios that are influenced by both P&L as well as RWA effects), it seems a clear deficiency that up to now,

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[10] Cf. e.g. EBA (2014a), p. 28 ff. in connection with ESRB (2014), p. 16 f. or Canabarro (2014), p. 55 for the US. Banks' internal stress test models often allow a more precise modelling of real estate prices with respect to different collateral object types (like single-family homes, multi-family homes, condominiums, …) or different local real estate markets and make use of intermediate variables.

neither regulators and banking practitioners nor researchers seem to have taken a closer look on the role of the loan-to-value distribution for stress testing LGD in mortgage loan portfolios. Since, as we will elaborate in this paper, a proper comparison of stressed LGDs across banking portfolios is in general not possible without adequate information about the differences between loan-to-value distributions, an answer to the above question seems a necessary prerequisite for any regulatory benchmarking. As far as we know, the existing literature on credit risk stress testing[11] has been completely silent on this problem.

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[11] Schuermann (2014), p. 7, notes that up to now there has been "very little discussion in the public domain" on how to arrive at stressed loan losses for the banking book in general. See also Foglia (2009) as well as the papers cited in footnotes 3 and 5. Calem/LaCour-Little (2004), pp. 661 ff. seem to have implicitly modelled the effect that we analyze as a part of their simulation study. For various different hypothetical portfolios that each consist of 500 single loans with their starting loan-to-value ratios uniformly fixed at either 80%, 90% or 95%, the authors re-sample quarterly paths of historical changes in regional house prices. They are able to show as one of their central results that risk-based capital requirements (in the sense of credit value-at-risk measures) strictly increase with their starting loan-to-value categories, but to a varying extent. However, they do not raise the issue of differing loan-to-value distributions.

3 Basic assumptions and definitions

We consider a portfolio of n loans that are collateralized by n assets, e.g. real estate objects, and we assume a one-to-one correspondence between loans and assets, i.e. each loan is collateralized by exactly one asset. For $i = 1, ..., n$ let L_i be the outstanding exposure of loan i and C_i, with $0 < L_i \leq C_i$, the corresponding collateral value at some point in time t.

Definition 1:

$$LTV_i := \frac{L_i}{C_i} \in (0,1]$$

is the (current) <u>Loan-to-Value-Ratio</u> (LTV) of loan i.

Hence, the Loan-to-Value-ratio is defined as the relation of the outstanding amount of the loan to the asset value of the property serving as collateral. We make two further remarks to this definition.

First, the collateral value C_i could either be inferred from observable market prices or it could be derived from a valuation model that is based on a discounted-cash-flow-approach. The latter kind of valuation is common in commercial real estate financing, where an independent appraisal is conducted to provide a mortgage value opinion.

Second, note that in reality, the LTV variable is not generally bounded at a value of 1. Conditions, under which LTV would exceed 1 even at the time of origination, could e.g. be created by a very aggressive risk taking behavior by the bank. The last crisis' most prominent example for such underwriting standards was the UK bank *Northern Rock*, whose home loan program offered residential real estate financing up to 125% of the collateral's current market price. Moreover, in several countries like the US, banks accept original LTVs above 80%[12] only, if a private mortgage insurance company is involved or if a similar protection is offered by a government-sponsored program.[13] Of course, current LTVs above 1 could result from a substantial drop in the collateral's market price during the time after origination that was higher than the meanwhile amortization, but this should be quite unlikely. Since we suppose that – without mortgage insurance – LTVs above 1 are rather a very rare exception than the

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[12] In the US residential mortgage market, the traditional maximum LTV is 80%, since a 20 % down payment is necessary to meet the Fannie Mae and Freddie Mac underwriting guidelines. See e.g. Qi/Yang (2009), p. 790.

[13] In an international setting, different private or public guarantee systems must be taken into account when comparing mean Loan-to-Value-Ratios across countries. See ECB (2009), p. 27 for an overview of the guarantee schemes in the Euro area.

rule in mortgage financing, we will admit this case only at some points in the paper for didactic reasons.

The <u>Loss Given Default</u> (LGD) of loan i in percent of L_i is defined as follows.

Definition 2:

$$LGD_i := \max\left[0, \frac{L_i - C_i \cdot RR_i}{L_i}\right],$$

where $RR_i \in [0,1]$ denotes the assumed <u>Recovery Rate</u> from the liquidation of the corresponding collateral asset in the case of the default of borrower i. The formula implies that upon default of the borrower there are no other proceeds than from the collateral assets.

In our notation, the recovery rate RR is a percentage figure that refers to the collateral value, but not to the loan amount.[14] It is calculated as the present value of the net liquidation proceeds of the collateral asset within some kind of foreclosure process that the bank initiates after a borrower's default, divided by the collateral value at the time of default. It is usually estimated by IRB-banks from their internal historical liquidation proceeds as a long-term average for each of the various object types (single-family homes, multi-family homes, condominiums, …).

In the next sections of the paper, the recovery rate will be subject to varying degrees of stress. The stress is modelled via a decrease in the recovery rate that is due to unfavorable market conditions. The approach is rather similar to the 2011 and 2014 *EBA* stress testing exercises, where the adverse stress scenario contains a set of given property price shocks. E.g. in the current 2014 *EBA* stress test a -8,8% drop in UK residential property prices for 2015 is assumed.[15]

Using both Definition 1 and Definition 2, we get the Loss Given Default for loan i as a function g of both its Recovery Rate and Loan-to-Value-Ratio:

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[14] Hence, the equation LGD = 1 − RR, which is very common in the literature and which has its origin in the LGD estimation for senior unsecured bonds, does not hold in our notation.

[15] Cf. ESRB (2014), p. 16. Moreover, it should be noted that this approach is quite similar to the concept of "downturn LGDs" under Basel II. However, the regulatory downturn LGDs may significantly deviate from stressed LGDs as less unfavorable market conditions are assumed under stress than for the regular RWA calculation. Furthermore, the stressed regulatory (downturn) LGDs may also deviate from the stressed point-in-time LGDs that are used not for the RWA-, but for the P&L projection.

$$LGD_i = g(LTV_i, RR_i) = \max\left[0, \frac{LTV_i - RR_i}{LTV_i}\right] = \max\left[0, 1 - \frac{RR_i}{LTV_i}\right] = 1 - \frac{RR_i}{\max[RR_i, LTV_i]} \quad .$$

The following figure shows the function g for different values of the Recovery Rate.
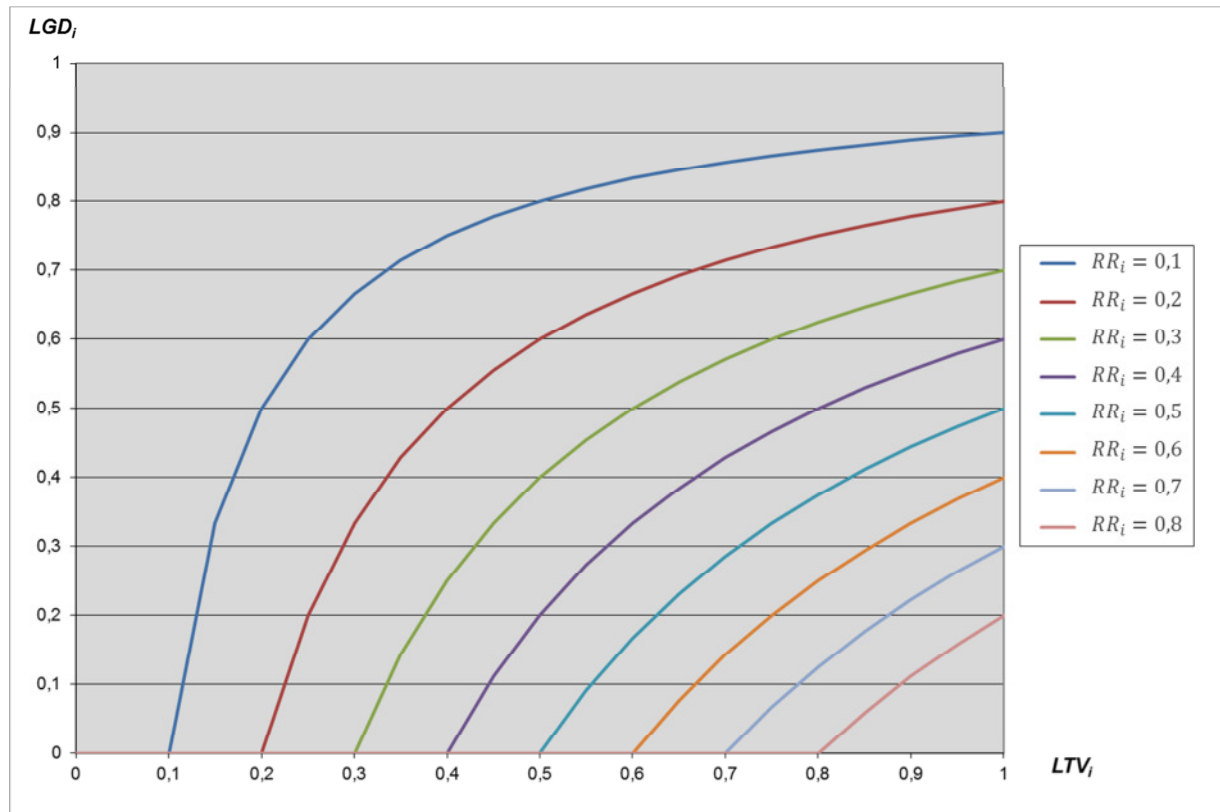
Figure 1: LGD_i **as a function of** LTV_i **and** RR_i**.**

According to the maximum operator in the formula above, LGD_i equals 0, if $RR_i \geq LTV_i$ holds. This simply means that there is no loss (but also no gain) for the bank, if the value of collateral exceeds the outstanding exposure. Because of the fundamental asymmetry between gains and losses that is characteristic for credit risk in general, the shape of the bank's payoff profile is capped and resembles in this respect that of a call option.[16] For $RR_i < LTV_i \leq 1$ and fixed RR_i, however, the function has a strictly concave shape. This concavity has no equivalent in option pricing theory.

Using the above definitions of LTV_i and LGD_i we are able to define the average Loan-to-Value-Ratio and Loss Given Default for a portfolio of n mortgage loans.

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[16] cf. Merton (1973).

Definition 3:

$$LTV_P := \frac{\sum_{i=1}^{n} L_i \cdot LTV_i}{\sum_{i=1}^{n} L_i} = \sum_{i=1}^{n} \frac{L_i}{\sum_{i=1}^{n} L_i} \cdot LTV_i \in (0,1]$$

is the mean exposure-weighted Loan-to-Value-Ratio of the portfolio P. It can be interpreted as an "average LTV per monetary unit", i.e. the expected LTV (in %) of a randomly chosen 1 EUR amount that is invested in the loan portfolio.

Definition 4:

$$LGD_P := \frac{\sum_{i=1}^{n} L_i \cdot LGD_i}{\sum_{i=1}^{n} L_i} = \sum_{i=1}^{n} \frac{L_i}{\sum_{i=1}^{n} L_i} \cdot LGD_i \in (0,1)$$

is the mean exposure-weighted Loss Given Default of the portfolio P. It can likewise be interpreted as an "average loss given default per monetary unit", i.e. as the expected loss (in %) of a randomly chosen 1 EUR exposure given the default of the respective borrower.

For the sake of simplicity, we will assume a constant recovery rate across all collateral assets in the portfolio considered and therefore drop the index i for the recovery rate in the following sections.

4   A simple numerical example

We begin our analysis by studying the simple numerical example that is given in Table 1 below. The aim of the investigation here is to clarify the economic role of the LTV-distribution in the case of a stressed recovery rate for the collateral assets.

Imagine the hypothetical Bank A that has a portfolio of only three residential mortgage loans of equal size, e.g. 250.000 EUR, that were handed out to three different borrowers, but each against the same amount of collateral. We assume that each of the three real estate objects serving as collateral has a current market value of about 400.000 EUR.

					Base Case		Stress Case	
				RR	60,0%		54,0%	
Bank A								
L_i		C_i		LTV_i	LGD_i (abs)	LGD_i (%)	LGD_i (abs)	LGD_i (%)
L_1	250.000	C_1	400.000	62,5%	10.000	4,0%	34.000	13,6%
L_2	250.000	C_2	400.000	62,5%	10.000	4,0%	34.000	13,6%
L_3	250.000	C_3	400.000	62,5%	10.000	4,0%	34.000	13,6%
Portfolio Total/Mean	750.000		1.200.000	**62,5%**	30.000	**4,0%**	102.000	**13,6%**

Table 1: Results for an identical LTV-Ratio across the loan portfolio (Bank A).

Then, by Definition 1, the LTV-ratio of each of the three loans is 62,5%. Further assuming a base case recovery rate of 60% of the current market value, when the borrower defaults within the upcoming year, and applying Definition 2, we receive an LGD of 4% for each loan. If we now move from this base case calculation to stressed conditions assuming – just like in the *EBA* 2011 stresstest example cited above – a minus 10% drop in housing prices that translates into a stressed recovery rate of 54%, the mean portfolio LGD according to Definition 4 rises from 4,0% to 13,6% by as much as 240%.

Let us now consider two other hypothetical Banks B and C, which differ from Bank A only in one respect, namely in that the total collateral value of 1.200.000 EUR is not equally distributed across the three loans. Bank B is constructed from Bank A by shifting 50.000 EUR of collateral value from Loan No. 1 to Loan No. 3 and Bank C is constructed from Bank B by simply shifting another 150.000 EUR from Loan No. 1 to Loan No. 3. Instead of shifting

collateral between the loans, one could of course alternatively change the exposure distribution to manipulate the three LTV-ratios.[17]

						Base Case		Stress Case	
					RR	60,0%		54,0%	

Bank B

L_i		C_i		LTV_i		LGD_i (abs)	LGD_i (%)	LGD_i (abs)	LGD_i (%)
L_1	250.000	C_1	350.000	71,4%		40.000	16,0%	61.000	24,4%
L_2	250.000	C_2	400.000	62,5%		10.000	4,0%	34.000	13,6%
L_3	250.000	C_3	450.000	55,6%		0	0,0%	7.000	2,8%
Portfolio Total/Mean	750.000		1.200.000	**63,2%**		50.000	**6,7%**	102.000	**13,6%**

Bank C

L_i		C_i		LTV_i		LGD_i (abs)	LGD_i (%)	LGD_i (abs)	LGD_i (%)
L_1	250.000	C_1	200.000	125,0%		130.000	52,0%	142.000	56,8%
L_2	250.000	C_2	400.000	62,5%		10.000	4,0%	34.000	13,6%
L_3	250.000	C_3	600.000	41,7%		0	0,0%	0	0,0%
Portfolio Total/Mean	750.000		1.200.000	**76,4%**		140.000	**18,7%**	176.000	**23,5%**

Table 2: Results for more dispersed LTV-distributions (Banks B and C).

Now, in terms of the mean portfolio LTV-ratio according to Definition 3, which can be used as a kind of traditional risk measure for residential mortgages, Bank B with a mean LTV-ratio of 63,2% appears only marginally more risky than Bank A with an original LTV-ratio of 62,5%, while Bank C with an LTV-ratio of 76,4% seems quite a bit more risky than Bank A and B under base case conditions. The reason for the observed increase in portfolio LTV is as follows: Since LTV is a strictly convex function in the collateral value C, the mean portfolio LTV is a strictly increasing function in the variance of the collateral distribution across the portfolio.[18]

When we compare the mean portfolio LGDs for the three banks A, B and C computed according to Definition 4 under the base case assumptions, it becomes apparent that the growing LTV dispersion also goes along with an increase in portfolio LGD. While this increase is quite dramatic for Bank C with a portfolio LGD of 18,7%, it is still rather substantial for Bank B with a portfolio LGD of 6,7% in comparison to the original LGD of Bank A, which is only 4,0%. The reason for the observed increase in portfolio LGD is that the

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[17] This spread of the collateral distribution is similar to the well-known concept of a "mean-preserving-spread" that was introduced into the theoretical literature on economic risk taking in the seminal Rothschild/Stiglitz (1970) paper. However, our approach to alter the LTV-distribution is not "mean-preserving" in the Rothschild/Stiglitz (1970) sense with respect to the LTV-distribution, but only with respect to the collateral distribution.

[18] This finding is already known in the literature on household leverage (see Korteweg/Sorensen (2013), p. 18) and is also a well-known result in the context of option pricing theory (see Jagannathan (1984), p. 429 f.), where it is attributed to Jensen's inequality.

overcollateralization on loan No. 3 that was achieved by reducing collateral for Loan No. 1 is economically worthless for the Banks B and C, given the 60% recovery rate assumption for the base case. Since a loan's LGD can on the one hand, by definition, not take on negative values, it does not reflect the available overcollateralization, while, on the other hand, a loan's LGD does reflect any existing undercollateralization. In other words, the non-linearity of the LGD function in LTV that is displayed in Figure 1 emphasizes the dependence of the mean portfolio LGD on the shape of the LTV-distribution.

When we now turn to a comparison of the mean portfolio LGDs for the three banks under the stress case assumptions, the results given in Table 2 above partly come as a surprise. While the stressed portfolio LGD is highest for Bank C with 23,5% as one would expect, it is with 13,6% now exactly the same for bank A and bank B. Obviously, the portfolio LGD of bank A and bank B does not differ any more under conditions of stress. The reason for this striking result is that the 50.000 EUR collateral that bank B holds more than bank A for loan No. 3 are worthless under normal conditions, but that they provide a valuable protection against the assumed housing price drop under stress. While under normal conditions, a 416.667 EUR current market value would have been sufficient to reduce the LGD of loan No. 3 to zero, a 462.963 EUR current market value for the collateral would be needed to reduce this loan's LGD to zero under stress.

More generally speaking, the simple three-banks-three-loans example indicates that the sensitivity of the portfolio LGD to stress in recovery rates decreases with an increase in the dispersion of the LTV-distribution. To see this, recall that while the assumed -10% drop in housing prices leads to an LGD increase for bank A by as much as +240%, this rise is much smaller for Bank B (+104%) and even further smaller for Bank C (+26%).

Further, the more stress is put on the portfolio via a lower assumed recovery rate, the less relevant the dispersion in LTV-ratios becomes. To demonstrate this effect, Table 3 provides the three banks´ mean portfolio LGDs for alternative recovery rates that range from 60% down to 41,7%.

LGD_P (%)	recovery rate RR				
	60,0%	55,6%	50,0%	45,0%	41,7%
Bank A	4,0%	11,1%	20,0%	28,0%	33,3%
Bank B	6,7%	11,1%	20,0%	28,0%	33,3%
Bank C	18,7%	22,2%	26,7%	30,7%	33,3%

Table 3: Portfolio LGDs for banks A, B and C for alternative recovery rates.

The economic reason for the displayed behavior is that the non-linear, asymmetric treatment of over- and undercollateralization in the single-loan LGD, which was explained above for the example of loans No. 1 and No. 3, looses importance when the collateral value diminishes. The influence of the differences in LTV-distributions on mean portfolio LGD vanishes completely, when the assumed recovery rate is equal to the minimum LTV of all the loans under consideration. Hence, the average portfolio LGDs of bank A and bank B converge to a single value of 11,1%, when the recovery rate crosses the minimum LTV of 55,6% (i.e. the LTV of loan No. 3 of bank B) and the average portfolio LGDs of all three banks converge when the recovery rates falls below 41,7% (i.e. the LTV of loan No. 3 of bank C, which is the overall minimum LTV among all nine loans). If – in the very extreme – the recovery rates would tend to zero, even the most excessive overcollateralization on a single loan would become as valuable for the bank as any other collateral, while the mean portfolio LGD reaches 100%.

We will now expand the analysis to cover some more real-world portfolios in chapter 5.

5 Simulation study for real-world portfolios

In this section, we aim to check the findings of the simple numerical example above for several more realistic mortgage loan portfolios. Since the LTV-distributions of banks' real mortgage loan portfolios are not publicly disclosed, so that there are no empirical data available, a simulation study seems the only way to numerically analyze the differing influence of housing market stress on portfolio LGDs.[19] For that purpose, we construct some hypothetical loan portfolios by performing Monte-Carlo simulations.[20] We model a multi-year process of portfolio generation by applying a set of rules that take into account loan origination, amortization and interest and by assuming a varying degree of risk appetite in banks' underwriting behavior (different mean LTVs for new mortgage deals) under different market conditions (different dispersion in newly originated mortgage LTVs).

5.1 *Methodology*

We extend the notation of Section 3 by adding a superscript t for the lifetime of the loan. So, the sequence $L_i^0, L_i^1, \dots, L_i^t, \dots$ stands for the exposure of loan i over time with L_i^1 denoting the exposure one period after its origination.

We further suppose that the bank issues 10 new loans with $L_i^0 = 100.000$ EUR each per month. While the fixed size of the loan amount seems a reasonable magnitude in housing finance[21], the number of newly originated loans is chosen arbitrarily. The number can be seen as a scaling factor for portfolio size and it has no material impact on the simulation results.

For each new loan i, its loan-to-value ratio at origination LTV_i^0 is drawn randomly from a uniform distribution between LTV_{min} and LTV_{max}. Hence, the unstressed collateral value $C_i = C_i^0 = C_i^t$, assumed to be constant for all t, is determined by

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[19] As far as we know, the rating agencies inquire LTV-distributions for banks' mortgage loan portfolios by using the same 10%-exposure-bucket approach that is commonly used in covered bond analysis. See e.g. the Finnish or the Spanish National Transparency Templates for covered bond data disclosure that is freely available under https://www.coveredbondlabel.com. However, such LTV-distributions are not disclosed by the rating agencies to the general public for banks' portfolios as a whole, but only by the banks themselves for the cover pools that underlie a specific covered bond issue.

[20] The only related article in the LGD literature that also uses Monte Carlo Simulation, and that we are aware of, is Calem/Latour-Little (2004), pp. 661 ff., who calculate economic capital by simulating regionally varying housing prices.

[21] Cf. e.g. ECB (2009), p. 37, who assume a standardized loan-balance of 100.000 EUR for cost comparison between different European countries.

$$C_i = \frac{L_i^0}{LTV_i^0}.$$

Since the majority of housing loans is granted with a long-term fixation of interest rates in the Euro-Area [22], we assume a fixed annual interest rate z_i that depends only on the initial Loan-to-Value ratio. Moreover, we use the same constant annual amortization rate $u_i = 1\%$ for all loans:

LTV_i^0	z_i	u_i
$\leq 60\%$	2,95%	1%
$\leq 70\%$	3,05%	1%
$\leq 80\%$	3,15%	1%
$\leq 85\%$	3,30%	1%
$\leq 90\%$	3,50%	1%
$\leq 95\%$	3,75%	1%
$\geq 95\%$	4,05%	1%

Table 4: Interest and amortization rate per annum used in the simulation.

While a 1% orderly amortization per year is rather typical for new loans in housing finance, the interest rates given in Table 4 are taken from a recent real world example.[23] The loan is paid back in constant monthly installments (annuity loan), so that amortization leads to a lower percentage share of interest in the installment over time. Such a redemption scheme is most usual in the vast majority of the countries in the Euro zone.[24]

The outstanding balance of each loan i that decreases every month t by amortization is calculated (ignoring compound interest issues) as:

$$L_i^t = L_i^{t-1} - \frac{\left(z_i \cdot \left(L_i^0 - L_i^{t-1}\right) + u_i \cdot L_i^0\right)}{12}$$

and

$$LTV_i^t = \frac{L_i^t}{C_i}.$$

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[22] Cf. ECB (2009), p. 25 f. According to this study, especially in Germany and France, about 85% of all housing loans are granted on a long-term fixed-rate basis with maturities of no less than 5 years.

[23] The interest rates used were taken on the 13.11.2013 from the website of the German subsidiary of Dutch ING Groep N.V. Table 4 displays the annual nominal interest rates for a 200.000 EUR housing loan with a 15 years maturity that were valid at that date. EMF (2013), p. 88 provides an overview about the evolution of representative interest rates on new residential loans for Europe and various other countries since 2001.

[24] See ECB (2009), p. 29.

Further, we remove at the end of each month all loans with a balance under 1.000 EUR from the portfolio assuming a complete repayment by the borrower. This is a purely technical rule that helps to avoid dealing with negative balances; it has no material impact on exposure-weighted LTV-distributions.

The simulation starts with a newly founded bank that just launches its business and therefore has no loans yet. The hypothetical residential mortgage portfolio of the bank grows over 600 periods, i.e. 50 years, following the rules described above. The reason why we stop the simulation at the end of this timeframe is simply that the portfolio reaches a steady state, in the sense that both the number of loans and the total exposure amount stay roughly constant. This equilibrium situation is due to the fact that the cash inflows from the amortizations and the cash outflows from newly originated loans roughly cancel out. In essence, this assumption goes along with the assumption of the bank having a well-established business model, so that there is no need to fund further growth.

Table 5 summarizes the input parameters of the six different portfolio simulations, which will be discussed in Section 5.2. The banks A1, A2 and A3 are conservative. They all issue new loans with a mean LTV-ratio of 60%. The three simulations differ only in the range of possible LTVs. Hence, the standard deviation varies roughly between 6% and 23%.[25] Banks B1, B2 and B3 are less conservative. Here, we assume identical dispersion parameters, but now with a mean LTV-ratio of 80%.

Bank	LTV-distribution for new loans			
	LTV_{min}	LTV_{max}	mean	stand. dev.
A1	50%	70%	60%	6%
A2	40%	80%	60%	12%
A3	20%	100%	60%	23%
B1	70%	90%	80%	6%
B2	60%	100%	80%	12%
B3	NA	NA	80%	23%

Table 5: Simulation input parameters for new loan LTV-ratios.

According to the available empirical evidence, mean LTVs of 60% and 80% seem to represent a reasonable range for modelling different degrees of risk aversion in banks' underwriting standards for residential mortgage loans. A 60% threshold applies for loans to

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[25] For the uniform distribution between a and b, mean and standard deviation are given by (a+b)/2 and $\sqrt{\frac{(b-a)^2}{12}}$.

be eligible as collateral for covered bonds in several European countries, e.g. in Germany.[26] This 60% threshold also goes along with reduced capital requirements under pillar I of the Basel II framework within the standardized approach. At the other end of the spectrum, the median LTV ratio for all new house purchase loans in the UK was up to 80% in 2007, the year when overall risk appetite was at its peak and the financial crisis began.[27]

As described above, we generally use uniformly distributed random numbers to generate different LTV-ratios at origination. The only exception to this approach is the simulation for bank B3, where we use random numbers following a beta-distribution with parameters $p = 1{,}6$ and $q = 0{,}4$ instead (cp. Section 6.1). The reason for the change in distribution is that we wanted to construct a portfolio with a standard deviation of the LTV-ratio for new loans that is almost as high as in the simulation for bank A3. However, because of the boundedness of the LTV-ratio between 0% and 100%, this is impossible using the uniform distribution.

The simulation script was written in R, version 3.0.1. Total runtime for each portfolio that involves drawing about 6.000 random numbers was about 40 minutes on a 2,4 GHz Intel Core i5 processor.

5.2   *Discussion of results*

Table 6 summarizes our simulation results for the six different banks described above. The size of the respective hypothetical portfolios lies around 5.300 loans and 330 million Euro exposure. As noted above, both these numbers would change with the number of newly originated loans per month as a (linear) scaling parameter. We calculated the portfolio LTV (LTV_P) and the portfolio LGD (LGD_P) according to the Definitions 3 and 4 in Section 3.[28] The resulting LTV-distributions are presented graphically later on in Figure 4.

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[26]   See ECB (2009), p. 29.
[27]   See EMF (2013), p. 72. As a result of the crisis, lenders tightened their underwriting standards, so that the median LTV in the UK came down to 75% in 2012.
[28]   For the standard deviation of (exposure-weighted) LTV in Table 6 we use the following formula:

$$\sqrt{\frac{\sum_{i=1}^{n} L_i \cdot LTV_i^2}{\sum_{i=1}^{n} L_i} - LTV_P^2}.$$

Bank	LTV new loans		simulation results				
	mean	stand. dev.	nr. of loans	total exposure	LTV_P	stand. dev. LTV	LGD_P $(RR = 60\%)$
A1	60%	6%	5.533	340.320.586	44,66%	13,64%	0,60%
A2	60%	12%	5.506	339.326.937	44,62%	15,69%	1,85%
A3	60%	23%	5.400	334.369.711	44,08%	21,73%	4,42%
B1	80%	6%	5.318	329.645.615	59,75%	17,57%	9,18%
B2	80%	12%	5.248	326.516.932	59,58%	19,22%	9,38%
B3	80%	23%	5.130	320.411.716	59,31%	24,92%	12,06%

Table 6: Summary of simulation results.

As a consequence of the two different means assumed for the LTV of newly originated loans, we essentially receive two different average portfolio LTVs. For the more conservative banks A1 – A3, who grant new loans with a mean LTV of 60%, the simulation yields a mean portfolio LTV of about 45%, whereas for the more aggressive banks B1 – B3, whose new loans start with a mean LTV of 80%, we end up with a portfolio LTV of about 60%. Clearly, the difference between the mean new loan LTV and the mean portfolio LTV at the end of the timeframe is due to amortization. Further, a connection between the assumed standard deviation of the new-loan-LTV and the standard deviation of the banks' portfolio LTV is apparent. Again because of the simulated amortization, the standard deviation of the portfolio LTV varies much less across banks than the assumed standard deviation of the new-loan-LTV.

The last column of Table 6 contains the portfolio LGD in a base case scenario, in which we assume a recovery rate of 60% for the collateral asset, just like in the simple example given in Section 4. As expected, the conservative banks A1 – A3 have distinctly lower portfolio LGDs than the less conservative banks B1 – B3. Moreover, a comparison of LGD within the two groups of banks yields that the mean portfolio LGD increases with the dispersion in the portfolio's LTV-distribution. We already noted a similar observation in Section 4 for the simple example.
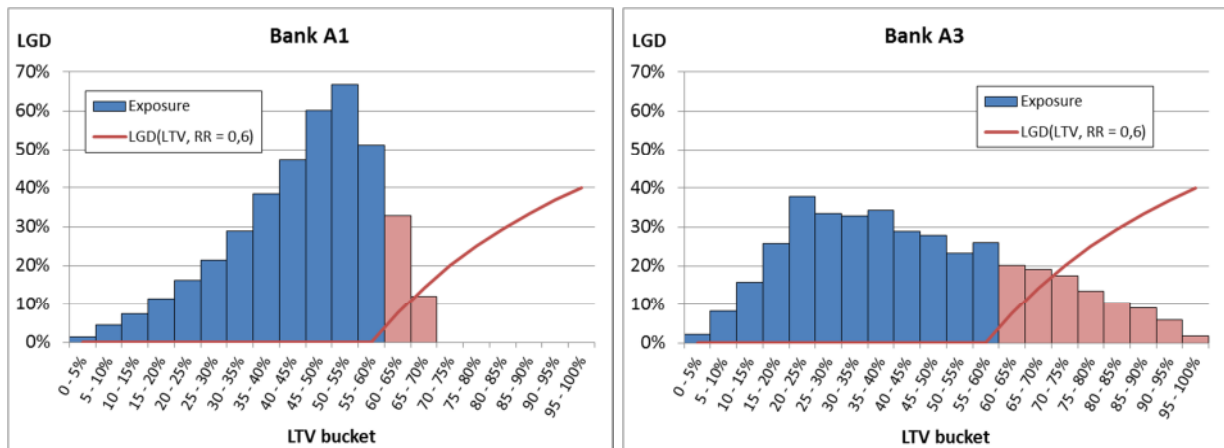
Figure 2: **LTV-distribution and LGD function for banks A1 and A3 in the base case scenario (RR = 60%).**

The reason for this effect is illustrated in Figure 2 by contrasting the LTV-distributions of bank A1 and bank A3. The red line displays the single-loan LGD as a function of LTV and recovery rate (cp. Figure 1 in Section 3) and the bar plot shows the exposure distributions for the two banks' portfolios with LTV buckets of 5% scaled in a convenient way. In the case of Bank A3, whose portfolio has a greater LTV dispersion, there is a considerably higher share of exposure falling into the range, where the LGD function is not capped at zero (red colored area). Hence, the proportion of exposure that causes real losses is much bigger, so that the portfolio LGD for bank A3 is more than 7 times greater (4,42%) than for bank A1 (0,60%) although the mean LTV of the two banks is almost identical. While this result again highlights the tremendous importance of the LTV-distribution for calculating portfolio LGD in general, it also confirms the finding of our simple numerical example in Section 4 under much more realistic assumptions.

What can be concluded from this analysis? Obviously, it is not a good idea to apply a "base case" mean portfolio LGD from one bank's portfolio as a benchmark to another bank, if the two banks differ substantially in their LTV-distributions. This result even holds true, if both banks' portfolios display almost identical mean portfolio LTVs. Of course, the argument can be generalized to benchmark parameters that are derived from peer groups built from a set of the banks that we consider. In this sense, it would be unjustified for a regulator to challenge e.g. an A1 bank that has a portfolio LGD of only 0,60% with a mean portfolio LGD parameter of 3.39% from a peer group that consists of two A2 and three A3 banks. In general, any comparison of mean portfolio LGDs for mortgage loans across banks that does not properly account for existing differences in LTV-distributions will not lead to reasonable conclusions, because it remains unclear whether observed LGD differences can be attributed to differences in LTV-distributions or are due to other factors, e.g. misspecified LGD models

or simply different recovery rate histories for banks' defaulted mortgage loans. Hence, regulators who perform benchmarking analyses on LGD differences[29] should be aware of this finding.

In the next step, we extent the analysis to cover varying degrees of macroeconomic stress. We assume a decline in housing prices that translates into stressed recovery rates of 50%, 40% and 30%.[30] Table 7 summarizes the resulting values of LGD_P for the six banks under consideration.

LGD_P	recovery rate RR			
	60%	50%	40%	30%
Bank A1	0,60%	4,78%	15,15%	30,72%
Bank A2	1,85%	6,17%	14,93%	29,72%
Bank A3	4,42%	8,96%	16,16%	27,40%
Bank B1	9,18%	18,82%	31,14%	45,67%
Bank B2	9,38%	18,36%	30,39%	44,88%
Bank B3	12,06%	19,98%	30,06%	42,66%

Table 7: Portfolio LGDs for alternative recovery rates.

By construction of the LGD formula (cp. Definition 2 in Section 3) the mean portfolio LGD increases with decreasing recovery rates. More interesting is the finding that the rank order of the banks within the two groups A and B according to their portfolio LGD turns around when the recovery rate goes down. In contrast to the normal situation, where more dispersion in the portfolio's LTV-distribution leads to a higher mean portfolio LGD, we can see here that under stress conditions more LTV dispersion may even go along with a lower portfolio LGD. The reason for that is again related to the characteristic shape of the LGD function consisting of both a cap and a concave component, which was discussed in Section 3. Figure 3 visualizes the different situations for the Banks B1 (low LTV dispersion) and B3 (high LTV dispersion) for the extreme case of a stressed recovery rate of 30%.

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[29] Cf. e.g. the benchmarking studies by BCBS (2013), EBA (2013) and EBA (2014b).
[30] Bearing in mind that defaulted residential real estate exposures tend to produce recovery rates of about only 50%-60% under normal market conditions, a recovery rate of about 30% of current market value seems quite unrealistic at first glance. If, however, we would assume a 35% drop in housing prices as recently announced by the Bank of England for their 2014 national stress test for UK banks (cf. e.g. http://www.bloomberg.com/news/2014-04-29/u-k-lenders-face-35-house-price-fall-in-boe-stress-test-1-.html) and we would start from a 50% recovery rate for defaulted exposures in the base case, we would indeed end up with a recovery rate of about 30%, more exactly with 32,5% = 50% x (1 − 35%).
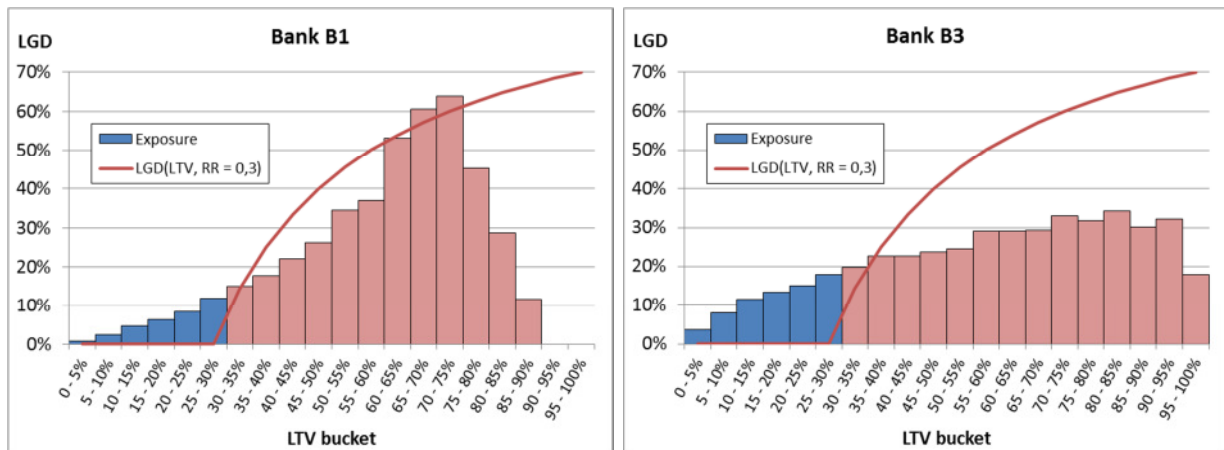
Figure 3: LTV-distribution and LGD function for Banks B1 and B3 in the extreme stress scenario (RR = 30%).

More specifically, there are two reasons for the lower portfolio LGD of Bank B3 under severe stress. First, the proportion of exposure with LGD equal to zero (blue-colored area) is bigger for Bank B3. Second, the widening of the LTV distribution in the concave range of the LGD function (red-coloured area) produces both more exposure with high LTV (80 – 100%) and more exposure with low LTV (30 – 45%). But due to the concave shape of the LGD function, the increase in low LGDs on the low-LTV-side exceeds the increase in high LGDs on the high-LTV-side. Therefore, the second effect leads to a lower portfolio LGD for bank B3 as well.

It is common practice in credit risk stress testing to state the percentage change of a parameter under stress in relation to its starting value under base case conditions with the help of stress factors as multipliers.[31] Table 8 entails these stress factors SF_P for the mean portfolio LGDs of the six hypothetical portfolios under the four different recovery rate levels.

SF_P	recovery rate RR			
	60%	50%	40%	30%
Bank A1	1,00	7,94	25,16	51,03
Bank A2	1,00	3,34	8,08	16,07
Bank A3	1,00	2,03	3,65	6,19
Bank B1	1,00	2,05	3,39	4,98
Bank B2	1,00	1,96	3,24	4,78
Bank B3	1,00	1,66	2,49	3,54

Table 8: Stress factors SF_P for alternative recovery rates.

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[31] E.g. the EBA followed such an approach in 2011, when benchmark parameters for PDs and loss rates were figured out by the ECB and then handed over to the participating banks.

The interpretation of the stress factors is straightforward: Supposing that a minus 16,67% drop in housing prices translates into a decline of the recovery rate from 60% to 50%, the mean portfolio LGD of Bank A1 would rise from 100% to 794% by as much as 694%.[32]

On the basis of Table 8, three important observations with respect to empirical stress factors can be made:

- The assumed differences in the six hypothetical banks' LTV-distributions lead in some cases to really huge differences with respect to the stress sensitivity of portfolio LGD. Those huge differences exist even when the portfolios' mean LTVs are almost identical, like e.g. within group A.

- The LGD stress factors are c.p. significantly higher for the conservative banks A1 – A3 than for their more aggressive counterparts B1 – B3.

- The stress factors decrease significantly with an increasing dispersion of the LTV-distribution, which acts as an additional risk buffer. This effect holds true within both groups A and B and is independent of the assumed recovery rate.

Consequently, the use of simple multipliers as stress factors for benchmarking purposes by regulators, rating agencies or banking practitioners is generally not reasonable for mortgage loan portfolios. While such a naïve benchmarking approach yields worthwhile results for senior unsecured bond LGDs, it will not work for mortgage loan portfolios that differ substantially in their LTV-distributions. Especially, those banks that have more dispersed LTV-distributions would be improperly disadvantaged by such a benchmarking approach, because it completely neglects the fact that they are equipped with a higher degree of over-collateralization, which becomes valuable under stress.

We will derive a formula which enables appropriate stress testing comparisons of mean LGDs for mortgage loan portfolios in the next chapter.

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[32] Of course, this percentage increase can also be directly computed from Table 7 as (4,78% / 0,60%) – 1.

6 Analytical LGD approximation for a beta-distributed portfolio LTV

In this section, we will first derive a closed-form solution for the mean portfolio LGD as a function of the recovery rate, which shall take into account the differences between empirical LTV-distributions (Section 6.1). To achieve that, we model a portfolio's LTV-distribution as a beta distribution, whose two parameters p and q convey the information about differences in its shape. Second, we will apply the formula to the six hypothetical portfolios A1-B3 by fitting the two parameters p and q via Maximum-Likelihood-Estimation and we will then compare the formula's results with the exact results from the previous section in order to assess the goodness of approximation (Section 6.2.).

6.1 *Methodology*

In order to arrive at a closed-form solution for portfolio LGD in the setting of our paper, a choice must be made as to the functional form of the LTV-distribution. Since there are no data on real-world LTV-distributions of banks' mortgage loan portfolios publicly available, we can base the assumption on the functional form solely on the six histograms from our Monte-Carlo-Simulations. From this empirical evidence, one can conclude that the LTV-distributions considered here are unimodal and not symmetric around the mean. Moreover, as already known from the discussion in Section 3, it seems reasonable to suppose that they are bounded between zero and one. One distribution function that fulfills these requirements is the Beta distribution, which is well-known in LGD-modelling and which is besides already used within the Basel II securitization framework.

So, let X be a beta distributed random variable on the unit interval $[0,1]$: $X \sim \beta(p,q)$, with $p > 0$ and $q > 0$. This variable represents the "LTV per monetary unit" as described in Definition 3. The probability density function of the beta distribution is defined by

$$\begin{aligned} f(x) &= \frac{1}{B(p,q)} x^{p-1}(1-x)^{q-1} \text{ for } x \in [0,1] \\ &= 0 \qquad\qquad\qquad\quad \text{ for } x \notin [0,1], \end{aligned}$$

where $B(p,q)$ is the beta function[33] and we denote the corresponding cumulative distribution function by

$$F_X(u) := \int_0^u f(x)dx \text{ for } u \in [0,1].$$

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[33]   Cf. e.g. Encyclopedia of Mathematics,
       http://www.encyclopediaofmath.org/index.php?title=Beta-function&oldid=14450.

Using the function $g(x) = 1 - \frac{a}{\max[a,x]}$ for some deterministic $a \in [0,1]$, we can derive another random variable Y on $[0,1]$ by

$$Y := g(X) = 1 - \frac{a}{\max[a,X]}.$$

Y can be interpreted as the "LGD per monetary unit" as in Definition 4. As $g(x)$ is a continuous function and $\int |g(x)| f(x) dx < \infty$ holds, the mean $E[Y]$, which corresponds to LGD_P in Definition 4, can be written as

$$E[Y] = E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) dx = \int_0^1 g(x) f(x) dx$$

and

$$\begin{aligned}
\int_0^1 g(x) f(x) dx &= \frac{1}{B(p,q)} \int_0^1 \left(1 - \frac{a}{\max[a,x]}\right) x^{p-1}(1-x)^{q-1} dx \\
&= \frac{1}{B(p,q)} \int_0^1 x^{p-1}(1-x)^{q-1} dx - \frac{1}{B(p,q)} \int_0^1 \frac{a}{\max[a,x]} x^{p-1}(1-x)^{q-1} dx \\
&= 1 - \frac{1}{B(p,q)} \int_0^1 \frac{a}{\max[a,x]} x^{p-1}(1-x)^{q-1} dx \\
&= 1 - \frac{1}{B(p,q)} \int_0^a x^{p-1}(1-x)^{q-1} dx - \frac{1}{B(p,q)} \int_a^1 \frac{a}{x} x^{p-1}(1-x)^{q-1} dx \\
&= 1 - F_X(a) - \frac{a}{B(p,q)} \int_a^1 x^{p-2}(1-x)^{q-1} dx.
\end{aligned}$$

Using the identity $\frac{1}{B(p,q)} = \frac{p+q-1}{p-1} \cdot \frac{1}{B(p-1,q)}$, which follows from properties of the beta function[34], and assuming $p > 1$, we further get the following closed-form solution for the mean portfolio LGD:

$$\begin{aligned}
E[Y] &= \int_0^1 g(x) f(x) dx \\
&= 1 - F_X(a) - a \cdot \frac{p+q-1}{p-1} \cdot \frac{1}{B(p-1,q)} \int_a^1 x^{(p-1)-1}(1-x)^{q-1} dx \\
&= 1 - F_X(a) - a \cdot \frac{p+q-1}{p-1} \cdot \left(1 - F_{X'}(a)\right),
\end{aligned}$$

where $F_{X'}(a)$ is the distribution function of a random variable X' with $X' \sim \beta(p-1,q)$.

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[34] Cf. e.g. Encyclopedia of Mathematics,
http://www.encyclopediaofmath.org/index.php?title=Beta-function&oldid=14450.

We can rewrite this equation in the notation introduced in Section 3 as

$$LGD_P = 1 - F(RR, p, q) - RR \cdot \frac{p + q - 1}{p - 1} \cdot \left(1 - F(RR, p - 1, q)\right),$$

with RR for the uniform recovery rate across all collateral assets in the portfolio.

Hence, we managed to express the mean portfolio LGD as an explicitly given function of the mean recovery rate on the portfolio's collateral and of the two parameters p and q that capture the LTV-distribution. The formula can be easily implemented within any spreadsheet environment, in which the cumulative distribution function of the beta distribution F is available, e.g. in Microsoft Excel.

6.2   *Discussion of results*

In order to estimate the two parameters p and q, which determine the shape of the LTV-distribution, for the six hypothetical bank portfolios we carried out the following Maximum-Likelihood-Estimation:

$$\sum_{i=1}^{n} L_i \cdot ln[f(LTV_i, p, q)] \to \begin{matrix} max \\ p,q \end{matrix} \quad ,$$

where f stands for the density of the beta distribution as introduced in Section 6.1.

The only deviation from the standard application of the Maximum-Likelihood-Method[35] is that we use the exposure amounts L_i as weighting factors for the log-transformed densities of the beta distribution.

The resulting density functions of the six fitted beta distributions are plotted as red lines against the LTV histograms already known from Section 5.2.

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[35]   Cf. e.g. Millar (2011).
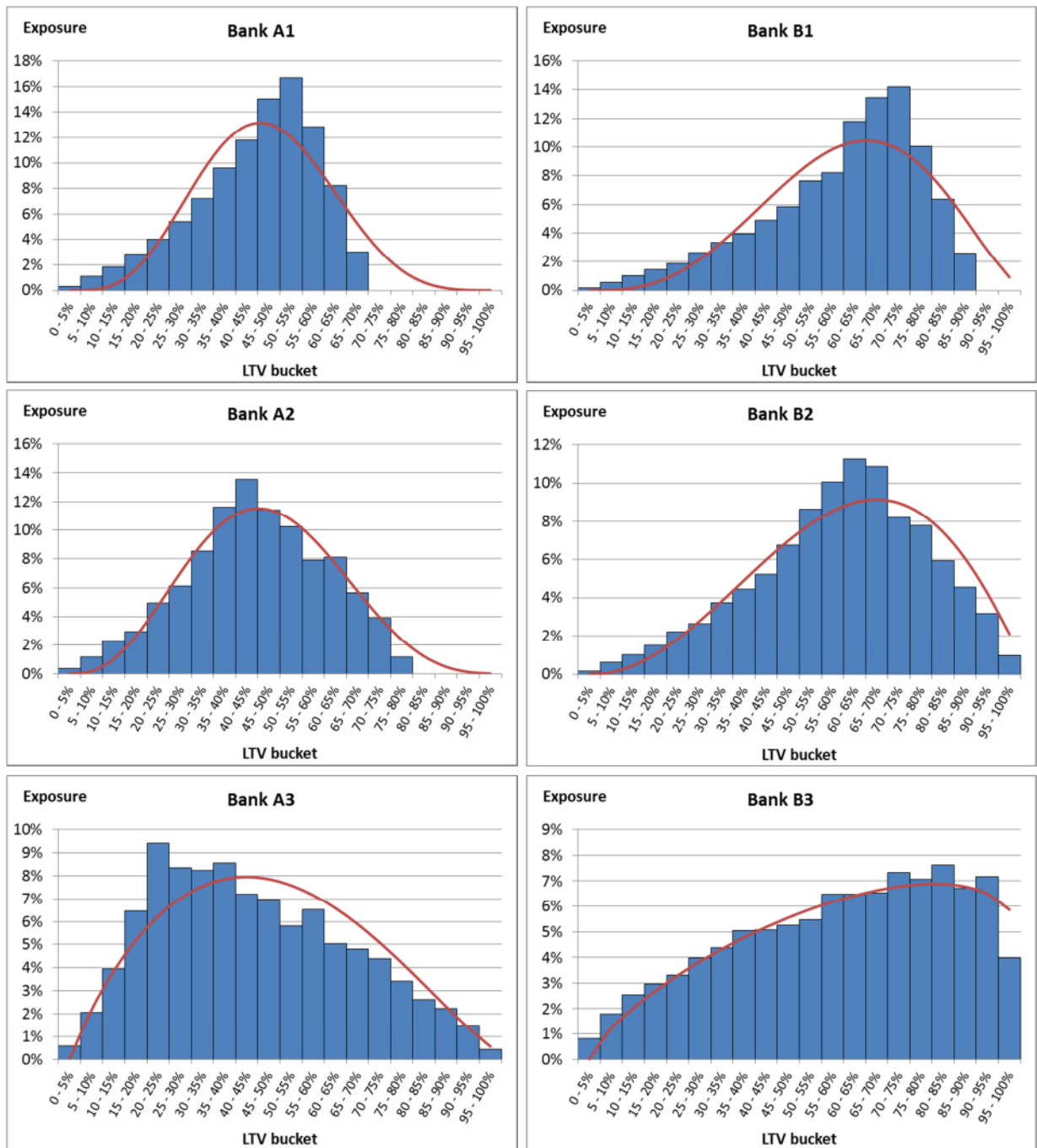
Figure 4: LTV-distributions for simulated portfolios and fitted beta distributions.

From a first optical inspection, the Maximum-Likelihood fit seems to work quite well for the six simulated LTV-distributions with bank A1 being the only clear exception. We will comment on this point later in the context of its consequences for mean LGD.

As a complement to Figure 4, Table 9 presents the estimated parameter values p and q as well as the resulting approximations for the mean and the standard deviation[36] of the LTV-distributions and contrasts these figures with the descriptive statistics for the Monte-Carlo-simulations, which are already known from Table 8.

Bank	simulation results		approximation by beta distribution					
	LTV_P	stand. dev. LTV	p	q	LTV_P	delta (%)	stand. dev. LTV	delta (%)
A1	44,66%	13,64%	4,95	6,24	44,24%	-0,95%	14,22%	4,27%
A2	44,62%	15,69%	3,85	4,83	44,34%	-0,62%	15,97%	1,80%
A3	44,08%	21,73%	1,93	2,39	44,63%	1,24%	21,55%	-0,80%
B1	59,75%	17,57%	4,11	2,85	59,04%	-1,19%	17,42%	-0,85%
B2	59,58%	19,22%	3,18	2,18	59,35%	-0,39%	19,48%	1,35%
B3	59,31%	24,92%	1,74	1,21	58,99%	-0,54%	24,77%	-0,60%

Table 9:   Descriptive statistics for the LTV-distributions: Approximation by beta distribution vs. simulation results.

The approximation produces mean LTVs that come pretty close to the mean LTVs of the loan-level data with a maximum percentage deviation of only about 1% for bank A3. The standard deviation of the LTV-distributions is reproduced almost equally well with bank A1 being a kind of an outlier with a percentage deviation of 4,27%. But, in terms of absolute deviations this is only a small error. So, on the whole, the Maximum-Likelihood fit of the beta distribution seems to work very well for capturing the first and second moments of empirical LTV-distributions.

We now turn to the following Table 10, which entails the six portfolios' mean portfolio LGDs from our formula and confronts them with the exact figures from the loan-level calculations already known from Table 7 (column "sim"). Again, we provide the percentage deviations between the exact approach and the approximation under both base case conditions (RR = 60%) and under varying degrees of stress (RR = 50%, 40%, 30%).

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[36] For the beta distribution, mean and standard deviation are given by p/(p+q) and $\sqrt{\frac{pq}{(p+q+1)(p+q)^2}}$.

LGD_P	recovery rate RR											
	60%			50%			40%			30%		
	sim	formula	delta (%)	sim	formula	delta (%)	sim	formula	delta (%)	sim	formula	delta (%)
Bank A1	0,60%	1,44%	-138,82%	4,78%	5,25%	-9,72%	15,15%	13,82%	8,75%	30,72%	28,70%	6,57%
Bank A2	1,85%	2,03%	-9,92%	6,17%	6,20%	-0,38%	14,93%	14,63%	2,04%	29,72%	28,68%	3,49%
Bank A3	4,42%	4,25%	3,93%	8,96%	9,11%	-1,69%	16,16%	17,01%	-5,25%	27,40%	28,89%	-5,47%
Bank B1	9,18%	8,52%	7,18%	18,82%	17,18%	8,68%	31,14%	29,36%	5,71%	45,67%	44,62%	2,29%
Bank B2	9,38%	9,63%	-2,58%	18,36%	18,07%	1,59%	30,39%	29,63%	2,50%	44,88%	44,17%	1,59%
Bank B3	12,06%	11,76%	2,50%	19,98%	19,51%	2,37%	30,06%	29,61%	1,50%	42,66%	42,31%	0,82%

Table 10: Mean portfolio LGDs for alternative recovery rates: approximation by analytical formula vs. simulation results.

In 23 out of the 24 comparisons shown in Table 10, the formula yields relative deviations in portfolio LGD of less than 10%. An accuracy of this magnitude seems to be absolutely sufficient for any type of LGD calculation, which is by its very nature already subject to estimation error. So, if the single exception in the case of bank A1 under a recovery rate of 60% would not exist, the goodness of approximation achieved by our formula would seem splendid. But, even in the case of bank A1, the absolute error in mean portfolio LGD produced by the formula is only 0,84%, which still should not lead to a vastly misleading impact on the bank's stressed P&L. To see this, recall that for a 10 billion EUR mortgage loan portfolio under a stressed PD of even 10%, the error in the bank's one-year P&L effect would be less than 10 Mio. EUR. This is because the mean LGD of bank A1 is extremely small.

Finally, Table 11 presents the stress factors for mean portfolio LGD that are calculated on the basis of a given p and q with the help of our formula and it also displays the percentage deviation from the exact results for the six hypothetical portfolios already known from Table 8.

SF_P	recovery rate RR								
	50%			40%			30%		
	sim	formula	delta (%)	sim	formula	delta (%)	sim	formula	delta (%)
Bank A1	7,94	3,65	54,06%	25,16	9,61	61,79%	51,03	19,96	60,88%
Bank A2	3,34	3,05	8,68%	8,08	7,20	10,88%	16,07	14,11	12,20%
Bank A3	2,03	2,14	-5,85%	3,65	4,00	-9,55%	6,19	6,80	-9,78%
Bank B1	2,05	2,02	1,62%	3,39	3,45	-1,58%	4,98	5,24	-5,27%
Bank B2	1,96	1,88	4,06%	3,24	3,08	4,95%	4,78	4,59	4,06%
Bank B3	1,66	1,66	-0,13%	2,49	2,52	-1,02%	3,54	3,60	-1,72%

Table 11: Stress factors for alternative recovery rates: approximation by analytical formula vs. simulation results.

With the exception of Bank A1, the approximation error is again rather moderate with a maximum percentage deviation of about 12%. Within group B, the multipliers resulting from our formula differ less than about 5% from those that result from the exact loan-level calculation. So, on the whole, the suitability of the formula seems quite promising with only the case of bank A1 slightly disturbing the positive picture. But, even with bank A1, one should bear in mind that a stress factor that is misspecified by about 60% would still yield the right order of magnitude for stressed portfolio LGDs and is certainly a better alternative than any naïve benchmarking attempt.

Before coming to the concluding chapter, we will discuss the approximation error of bank A1 here in more detail: The main reason for the distorted mean portfolio LGD is that we assume a beta-distributed LTV variable on the whole interval [0; 1]. In the case of bank A1, however, the empirical LTV-distribution is restricted to the interval [0; 0,7]. Due to the functional form of the density, which implicitly assumes that there is some positive probability mass above an LTV of 70%, the Maximum-Likelihood fit cannot yield a good approximation of the empirical distribution, which is also reflected in the overestimated standard deviation mentioned beforehand in the context of Figure 5. As a consequence, the fitted density, which is inevitably too high for LTVs above 70%, leads to a significant overestimation of portfolio LGD in the base case with a 60% recovery rate. As can be seen from Table 10, this misspecification loses importance with the stress level, so that the approximation error diminishes to a normal extent in the other recovery rate scenarios. However, the error in the anchor scenario infects the stress factors in Table 11 for all other recovery rate levels. In order to finally judge the practical relevance of this type of approximation error, it is necessary to analyze real-world mortgage loan portfolios. If these – other than bank A1 in our simulation study – all contained at least some LTVs above the 70% level, we would just by chance have constructed with bank A1 a pathological case, which does not endanger the accuracy of our formula in practice.

7   Conclusions and outlook

In this paper, we showed that the impact of stress on the mean LGD of a mortgage loan portfolio depends heavily on the shape of the underlying LTV-distribution. Furthermore, we demonstrated how the impact of a pre-specified drop in collateral values on portfolio LGD can be properly modelled for banks' portfolios of mortgage loans that differ in their loan-to-value distributions. With a certain collateral value decline taken as given, the calculation of an average stressed LGD for a mortgage loan portfolio is rather straightforward, when loan-level data are available. However, since loan-level calculations are not feasible for regulators or rating agencies, it is an open question how the resulting portfolio LGDs that play a vital role in current stress testing frameworks can be verified without knowledge of the complete LTV-distribution. In particular, it is unclear, how comparisons of mean LGDs across banks should account for differences in the portfolios' LTV-distributions and how these differences can be incorporated in a suitable benchmarking framework.

In order to resolve these issues, we propose a closed-form solution for the mean LGD in a mortgage loan portfolio that seems to enable meaningful comparisons without recourse to loan-level data. The suggested formula is based on the assumption that loan-to-value is beta-distributed and it can be easily implemented in a spreadsheet environment. The idea of the approximation is that a two-parameter fit of a portfolio's loan-to-value distribution will contain sufficient information about the structure of overcollateralization to assess the effect of stress on mean portfolio LGD. If banks were required to provide a maximum-likelihood estimation of these two parameters for their mortgage loan portfolios, regulators and rating agencies would be placed in a position to independently assess the plausibility of banks' stressed LGDs under various stress scenarios. Moreover, the formula may also be used for benchmarking regulatory LGDs that advanced IRB-banks produce under pillar I of Basel II.

Since the loan-to-value distributions of banks' mortgage loan portfolios are currently not disclosed to the public, we estimated the parameters for several hypothetical portfolios and checked the robustness of the formula's results with the help of a simulation study. The formula seems to yield sufficiently accurate approximations for most cases. Future research could try to identify alternative closed-form solutions for other density functions, especially those, which are also able to capture an extremely leptokurtic behavior. However, in order to assess the general adequacy of our approach in practice, it would be necessary to implement the maximum-likelihood estimation for a set of real-world bank portfolios. Apart from real estate mortgage loans, the methodology could also be applied in all other kinds of collateral-based lending, where loan-to-value considerations play an important role, e.g. in shipping and aviation finance.

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